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ELEMENTARY TREATISE

ON

NAUTICAL ASTRONOMY.

FOR THE USE OF

SCIENCE CLASSES AND SEAMEN.

BY

HENRY EVERS, LL.D.,

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AUTHOR OF "NAVIGATION," "LAND AND MARINE ENGINE," "LOCOMOTIVE ENGINE," ETC.

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PREFACE.

We know of no previous attempt to simplify Nautical Astronomy, or of any attempt whatever to give the rudiments of the science by themselves for the benefit of beginners. It is therefore hoped that this effort will be viewed with a little favour in consequence. It will be found that the simpler parts of Nautical Astronomy are not so abstruse after all, if the learner will but give a little time, patience, and attention to the study. The Author is aware that he does not address himself to a very large audience, yet hopes that this small volume may be the means of inducing many to read what has hitherto been considered a difficult subject.

H. E.

Plymouth, *May 1873.*

CONTENTS.

CHAPTER VIII.

Longitude by Chronometer.

CHAPTER IX.

Amplitudes and Azimuths.

CHAPTER X.

The Moon.

NAUTICAL ASTRONOMY.

CHAPTER I.

Definitions—Poles—Celestial Equator—Circles of Declination, or Hour Circles—Six o'Clock Hour Circle—Parallels of Declination—Zenith—Nadir—Horizon (Sensible, Rational, and Visible)—Circles of Altitude—Azimuth or Vertical Circles—Celestial Meridian—Prime Vertical—Altitude—Declination—Polar Distance—Right Ascension—Right Ascension of Meridian—Ecliptic—Circles of Latitude—Latitude and Longitude—Obliquity of Ecliptic—Equinoctial Points—Signs of the Zodiac—Equinoxes—Precession of the Equinoxes—Shape of Earth—Axis—Poles—Equator—Equatorial and Polar Diameters—Compression—Proofs of Earth's Sphericity.

1. Definitions.—Nautical Astronomy is the application of astronomical facts and deductions to nautical purposes. It is a science founded on observation and astronomical data, applied to the purposes of the navigator.

Nautical Astronomy is a science which treats of the positions of the heavenly bodies on the celestial concave, and from their relative position it gives the mariner rules to find his on the earth's surface.

It has been suggested that Navigation proper should be called *Geo-navigation*, and Nautical Astronomy *Celo-navigation.* They would certainly be more exact terms for the student, but might prove a little difficult to the sailor.

The position of a heavenly body is generally referred

to two great circles drawn on the celestial concave, the celestial *equator*, and the *ecliptic*.

2. **Poles of the Heavens.**—If the axis of the earth be produced both ways to meet the heavens, it would indicate the position of the celestial poles. Could a person stand at the north pole of our earth, the north pole of the heavens would be exactly over his head.

The whole celestial concave appears to turn upon the two poles as upon two points. Every circle of the sphere parallel to the celestial equator has one of these points for its pole, every point in such circle being equi-distant from it.

3. **A Great Circle** is one that divides the sphere into two equal parts.

4. **Celestial Equator.**—The plane of the terrestrial equator produced to the sky shows the celestial equator; or, the *celestial equator* is a circle described on the heavens by the plane of the earth's equator produced. It is called the *equinoctial*.

5. **Circles of Declination,** or **Hour Circles,** are great circles of the celestial sphere passing through the poles of the heavens, and cutting the equator or equinoctial at right angles. They are the planes of our meridians extended to the heavens.

6. **Six o'Clock Hour Circle** is the circle of declination which passes through the east and west points of the horizon.

7. **Parallels of Declination** are small circles parallel to the celestial equator, passing through all points of the celestial sphere that have the same declination. They are the planes of the parallels of latitude extended to the heavens. Thus it will be perceived that *declination on the celestial sphere corresponds to latitude on the earth.*

8. **Zenith** is the point of the heavens exactly over the observer's head.

9. **Nadir** is the point in the celestial concave exactly opposite the observer's feet. It is therefore diametrically opposite the zenith.

10. Horizon.—We have the *sensible* horizon, the *rational* horizon, and the *apparent* or *visible* horizon.

11. Sensible Horizon.—The sensible horizon is shown by a plane touching the earth at the spectator's feet, and then extended to the celestial concave.

12. Rational Horizon.—Rational horizon is shown by a plane through the centre of the earth drawn parallel to the visible horizon, and cutting the sky.

13. Visible Horizon.—The apparent or visible horizon is where the sky and earth appear to meet, forming a circle round the spectator, of which he is the centre.

14. Circles of Altitude—Azimuth or Vertical Circles.—A circle of altitude is a great circle of the celestial sphere, passing through the zenith and nadir, and cutting the horizon at right angles. They are also called azimuth or vertical circles.

15. Parallels of Altitude are less circles of the sphere parallel to the horizon, passing through all objects that have the same altitude.

16. Celestial Meridian, or **Meridian of the Observer** is the *circle of declination* passing through the poles of the heavens and the zenith and nadir of the observer, and consequently the north and south points of the horizon. This vertical circle sweeps daily over the heavens from west to east, but, of course, it *appears* fixed, and all the celestial bodies seem to pass over it once a day.

17. Right Ascension (R. A.) of an Object is the arc of the equator intercepted between the first point of Aries and the circle of declination passing over that object; or R. A., for so right ascension is abbreviated, is the angle at the pole between two hour circles, one passing through the first point of Aries, the other over the place of the object.

18. Right Ascension of the Meridian is the arc of the equator intercepted between the first point of Aries and the celestial meridian. It is reckoned eastward, following the order of the signs of the zodiac.

19. Azimuth.—The azimuth is the angle at the

between two circles of altitude, one passing through the north and south points of the horizon, the other over a celestial object; or it is the arc of the horizon lying between the south point in north latitude and the north point in south latitude, and a circle of altitude passing through any celestial body.

20. Amplitude of a heavenly body is its distance from the east point when rising, and west point of the horizon when setting.

21. Ecliptic is the circle which the sun appears to describe annually among the fixed stars; or it is the apparent path of the sun in the sky. It is therefore the true equator of the heavens. No *eclipse* can take place unless the moon is in the ecliptic, hence its name.

22. Circles of Latitude are great circles of the celestial concave passing through the poles of the ecliptic, and therefore cutting the ecliptic at right angles.

The *Latitude* of a heavenly body is the arc of a circle of latitude intercepted between the body and the ecliptic.

The *Longitude* of a celestial object is the arc of the ecliptic, intercepted between the first point of Aries and a circle of latitude passing over the body.

Note, therefore, that celestial latitude and longitude refer to the *ecliptic,* and not to the equator.

The angular distance of the hour circles is reckoned from the first meridian westwards.

23. Prime Vertical is that circle of altitude passing through the observer's zenith and nadir and the east and west points of the sky, its plane is, therefore, at right angles to that of the celestial meridian. It is called *prime* because the angular distances at which bodies rise or set are measured from it; an object rising exactly on it at the equator will describe a great circle in the sky perpendicular to the horizon.

24. Altitude of an Heavenly Body is the angular distance of that body from the horizon measured on a vertical circle, its *complement* is called the **zenith distance.**

Other definitions of altitude are given farther on, where we treat of altitudes.

25. Zenith Distance is the angular distance of an object from the observer's zenith, measured on a circle of altitude.

26. Declination of a celestial object is its angular distance from the celestial equator. Its complement is the *polar distance.*

27. Polar Distance is the angular distance of an object from the pole, and does not correspond to terrestrial polar distance; *right ascension* and *declination correspond* to latitude and longitude on the terrestrial sphere.

28. Obliquity of the Ecliptic is the inclination of the ecliptic to the equator, or it is the angular distance between the pole of the ecliptic and the pole of the equinoctial. The angle at which the ecliptic inclines to the equator for January 1, 1874, is 23° 27′ 20″·41, but varies a little. The obliquity of the ecliptic is the cause of the seasons; for it follows that the axis of the earth must *be* inclined to the plane of her orbit at an angle of 66° 32′ 40″.

29. Equinoctial Points are the two points in which the ecliptic cuts the equator. They are called the first point of Aries and first point of Libra.

The Zodiac is a zone extending about 8° on each side of the ecliptic, and is divided into 12 divisions, called the signs of the zodiac.

30. The Signs of the Zodiac are twelve:—

Ram *Aries*	Bull *Taurus*	Twins *Gemini*	Crab *Cancer*
Lion *Leo*	Virgin *Virgo*	Scales *Libra*	Scorpion *Scorpio*
Archer *Sagittarius*	He-goat *Capricornus*	Waterman *Aquarius*	Fishes *Pisces*

These are easily remembered from the following rhyme:—

The *ram*, the *bull*, the heavenly *twins*,
And next the *crab* the *lion* shines,
The *virgin* and the *scales*,
The *scorpion*, *archer*, and *he-goat*,
The man that bears the *watering pot*,
And *fish* with glittering scales.

31. The Equinoxes are the periods of the year in which the days and nights are equal all over the world. There are two equinoxes, the Vernal in Spring, and Autumnal in Autumn, the former happening about March the 20th, and the latter about the 22nd of September; these are the two dates on or near which the sun crosses the celestial equator, when its declination for an instant vanishes, and changes from south to north in March, and from north to south in September.

32. The Precession of the Equinoxes is the retrograde motion of the equinoctial points along the ecliptic at the rate of $50''\cdot224$ per annum. The equinox goes backwards from east to west annually to meet the sun the space of $50''\cdot224$, so that a complete revolution will be effected in 25,868 years. Each year the equinox *precedes* the previous one by about 3 minutes 20 seconds. It may be illustrated thus: it is found that the latitude of a star never alters, it is constant; but the longitude alters every year, increasing 50·224 seconds of arc. The vernal equinox coincided four thousand years ago with the constellation of stars named Aries; but on account of the uniform regression along the ecliptic of the equinoctial point, this point is now moved backwards into the constellation Pisces; but it is still customary in astronomical, nautical, and mathematical works to call the vernal equinox the first point of Aries.

Hitherto no reference has been made to the shape, size, etc., of the earth; as these have an important bearing on the science of nautical astronomy, they must not be omitted.

33. The Shape of the Earth is an oblate spheroid, or a sphere slightly flattened at the poles.

34. Axis.—The axis of the earth is an imaginary line upon which it is supposed to turn once in twenty-four hours.

35. Poles.—The ends of the axis are the north and south poles respectively.

36. The Equator is a great circle drawn round the middle of the earth at equal distance from the two poles.

We have already presumed that the student is thoroughly well acquainted with the four preceding definitions, but they are repeated for convenience.

37. The Equatorial Diameter is a line passing through the centre of the earth, terminated at both ends by the equator.

38. The Polar Diameter coincides with the axis of the earth.

These two diameters are not of the same length. Professor Airy has calculated them, and gives their lengths as under:

Equatorial	diameter	is	7925·648	miles	long.
Polar	,,	,,	7899·170	,,	,,
Average	,,	,,	7912·409	,,	,,

Bessel, the great Prussian astronomer, gives them thus:

Equatorial	diameter	is	7925·604	miles	long.
Polar	,,	,,	7899·114	,,	,,
Average	,,	,,	7912·309	,,	,,

According to Airy the equatorial diameter is longer than the polar by 26·478 miles, which is obtained from the above by simple subtraction. Bessel's calculation will give the difference 26·49 miles; both calculations being made independently, it is astonishing how nearly they coincide, which fact shows the closeness and correctness of the reasoning employed in each case.

39. Compression.—The ratio of the difference between the polar and equatorial diameter to the equatorial diameter is called the *compression.*

From Professor Airy's calculation the

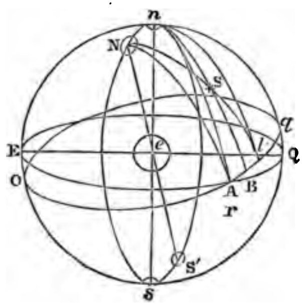
$$\text{Compression} = \frac{26{\cdot}478}{7925{\cdot}648} = \frac{1}{299{\cdot}69} = \frac{1}{300} \text{ nearly.}$$

From Bessel's the

$$\text{Compression} = \frac{26{\cdot}49}{7925{\cdot}604} = \frac{1}{299{\cdot}19} = \frac{1}{300} \text{ nearly.}$$

It is seen that both give the same result, viz., the compression is $\frac{1}{300}$ part of the equatorial diameter.

40. Illustrations of Definitions.—Let the annexed figure be a perspective representation of the celestial sphere; *e* the earth.



Let E A Q represent the celestial equator, while its poles are at *n* and *s*. Let O A *q* represent the ecliptic, while its poles are at N and S′. A, or where the equinoctial and the ecliptic intersect, is the first point of Aries, generally represented by the sign ♈.

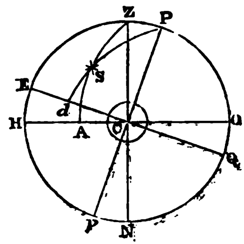
If we suppose S to be the place of the sun, a star, or other celestial object, we can now show what is meant by its latitude, longitude, R. A., declination, polar distance, etc.

AB is the R. A. of S, or angle A*n*B is the right ascension. S B is the declination of the object S. A*l*, or angle A*n* *l*, is the longitude, and S*l* is the latitude of a celestial object situated at S. Since S B is the declination, the polar distance is S*n*.

Next figure will show the *altitude, zenith, nadir, zenith distance, polar distance, declination, azimuth, etc.*

Let this be a projection of the sphere on the plane of the meridian P E *p* O. The point C is either the east or west point of the horizon, H the south, and O the north. H O is the horizon of the observer; P*p* is the axis of the

heavens; E Q is the equinoctial; P is the pole; Z is the zenith of the observer; N is the nadir.

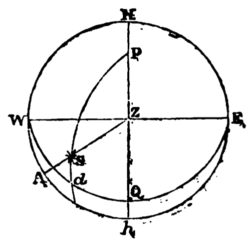


P p will represent the six o'clock hour circle, while Z N will represent the prime vertical.

Let C be the centre of the earth and sphere. Let S be the position of any celestial object. Then S A is its *altitude;* then S Z is its zenith distance; S d is its declination; and S P is its polar distance.

The angle Z P S or E P d is the hour angle, the distance of the object from the meridian. The angle H Z A is the azimuth.

There is yet another mode of illustrating these positions of an heavenly object.



Let this be a projection of a sphere on the plane of the horizon, or on the supposition that the eye of the observer is at his zenith looking directly down on to the centre of the earth. H W A E is the horizon, Z the zenith, H Z h is the meridian, and W Z E the prime vertical. Z S A is a circle of altitude or azimuth circle; P is the pole, and let S be the position of an object; then A S is the altitude of the object; S Z is the zenith distance of the object; S P is the polar distance of the object; and if W Q E be the celestial equator, then S d is the declination of the object; S P Z is the hour angle of the object; P Z S is the azumith of the object.

41. The Earth is a Sphere.—The chief illustrations to show the rotundity of the earth are these:—

(*a*) A person standing close to the water on the sea-shore, and watching the approach of a ship under sail,

would first, with a good telescope, see the topmast and topsail, then the lower sails, next the deck and the hull; whilst, were the ship leaving the land, he would first lose sight of the hull, then the lower sails and yards, next upper sails and upper yards, and, lastly, the topmast. So, if two ships at sea catch a sight of each other, and then sail towards the same point in the horizon, or meet, the same appearance is presented; first the mast heads appear in sight to each other, then, in their order, the upper sails, lower sails, and hull. Were the earth a flat plane, the very opposite to the above would take place; for the hull, being the *largest*, would first appear to view, or would be the last to disappear, while the smaller masts and sails would be the first to disappear in the distance, or the last to come into sight as the vessel approached the spectator.

(*b*) Ships, by keeping constantly on a westerly or easterly course, have returned to the same place again; *i.e.*, they have set sail say from the Cape of Good Hope, and have sailed westerly round Cape Horn, held on the same course by Australia, and have come across the Indian Ocean from the east, finally reaching the Cape again. This takes place in every direction on the earth's surface, therefore it is a globe.

(*c*) As we proceed from any given place north or south, new constellations in the heavens continually make their appearance, while others sink from view. This can only take place on the supposition that the earth is round from north to south. It is the same shape from east to west, or else a watch correct in London would be correct at Constantinople or New York, and would show true time wherever it was carried; but we know that as we go towards the east, the watch is too slow, and if towards the west, too fast. We are well aware that this fact proves that the earth moves from west to east; but it also proves that the earth is a sphere, for if it were a plane, the sun would *light up the whole surface at the same instant,* or as soon as the plane turned to-

wards the sun, and all clocks would indicate the same time.

(*d*) In making trigonometrical surveys, it is constantly necessary to lay down large triangles on the surface of the earth, and when the three angles of these triangles are measured by proper instruments, it is invariably found that their sum is greater than three right angles, *i.e.*, they are therefore spherical triangles and lie on a spherical surface; therefore we are at liberty to take this circumstance as an additional proof of the sphericity of the earth.

(*e*) When an eclipse of the moon takes place, the shadow of the earth is projected upon the moon. This shadow is always circular, therefore the earth is a sphere.

(*f*) Arcs of great circles of the earth have actually been measured.

There are other proofs of the earth's sphericity, but the above are sufficient for present purposes.*

* The teacher is recommended to obtain *Glossary of Navigation*, by the Rev. J. B. Harbord, where he will obtain a capital insight into all definitions connected with navigation and nautical astronomy.

CHAPTER II.

THE DAY AND TIME.

Measure of Time—Mean and Apparent Time—Equation of Time —Apparent and Mean Noon—Mean and Apparent Solar Day—Sidereal Day—The Year—Civil and Astronomical Time—Time and Arc—To Convert Arc into Time and Time into Arc—Examples—Rules—Greenwich Date—Examples.

42. The Mean Sun.—The earth moves on its orbit round the sun at varying velocities; at one time, in winter, in the northern hemisphere, moving much quicker than at another. It moves at its minimum velocity when it is summer in our hemisphere. The sun, in its rising, its meridian altitude, and in its setting, is the great measurer of time, it also regulates the return of the seasons and sets their bounds; but because the sun moves irregularly (in reality the earth), his measures of time are a little defective; therefore, as astronomers would be put to very great inconvenience if they had no correct marker of equal intervals of time, they have had to devise means to compensate for the sun's irregularities. The means provided to carry out this object is an imaginary sun, called the *mean sun*, which is supposed to move uniformly along the equator at a daily rate of $59' \, 8''\cdot 33$ of arc, while the *real sun*, with his irregular motion, passes along the ecliptic. Because the earth moves quicker at one time than another (the student must carefully observe that we are speaking as if the *sun* moved, and not the earth), the *mean* sun will sometimes come to any given meridian before the *real* or *true* sun, and sometimes after it.

43. Equation of Time.—The difference between the

times that the *real* and *supposed* sun come to the meridian is called the equation of time, or equation of time is the angle at the pole contained by two circles of declination passing over the centres of the *true* and *mean* suns. It enables us to reduce apparent to mean time, or mean time to apparent time.

44. **Apparent Noon.**—When the true sun is exactly on the meridian it is apparent noon.

45. **Mean Noon.**—When the fictitious, or mean sun, is exactly on the meridian it is mean noon.

46. **Sidereal Noon** is when the first point of Aries is on the meridian.

47. **A Day** is the interval between two successive transits of a celestial body across the same meridian.

48. **Mean Solar Day.**—A *mean solar day* is the interval between two successive transits of the mean sun over the same meridian.

49. **Mean Time** is time measured and indicated by the mean sun. It may also be defined as the angle at the pole included between the meridian and the circle of declination passing over the place of the mean sun.

50. **Apparent Solar Day.**—An *apparent solar day* is the interval between two successive transits of the sun over the same meridian.

51. **Apparent Time** is time measured and indicated by the apparent or true sun. It may also be defined as the angle at the pole included between the meridian and the circle of declination passing over the place of the true sun.

52. **Sidereal Day.**—A *sidereal day* is the interval between two successive transits of the same star over the meridian, or the transit of the first point of Aries over the meridian.

53. **Sidereal Time** is the angle at the pole between the meridian and the circle of declination passing over the first point of Aries.

54. **Mean Solar Year,** or *Mean Tropical Year,* is the interval that elapses between the sun leaving the first point of Aries, and returning to it again. Because the

first point of Aries has a slow retrograde motion, and the motion of the sun in the ecliptic is subject to irregularities, solar or tropical years vary slightly.

55. Sidereal Year is the interval elapsing between the sun leaving any certain fixed star, and returning to the same again.

We have two modes of reckoning time: (1) the Civil; (2) the Astronomical.

56. A Civil Day we reckon as beginning at one mid-night, and ending at the next mid-night.

57. An Astronomical Day begins at one noon, and ends at the next noon; the astronomer also numbers his hours from 0 to 24.

Thus the difference between the civil and astronomical mode of reckoning time is, that in the former the day is supposed to commence immediately after 12 P.M., while in the latter it does not begin until noon, and runs the 24 hours through. Civil and astronomical time agree when the former is P.M. time; but to obtain the latter from the former when it is A.M. time, it is necessary to add 12 hours to the given time, and take the day before. Thus, if civil time be May, 17th day 10h. 30m. P.M., the astronomical time is also May, 17th day 10h. 30m.; but if the civil time be May, 18th day 2h. 40m. 11 sec. A.M., the corresponding astronomical date would be May, 17th day 14h. 40m. 11sec. The distinction will be pointed out more clearly as we proceed.

58. Time and Arc.—As the earth turns round once in 24 hours, every one of the 360°, into which all circles are divided, will pass the sun in the 24 hours;

$$\therefore \text{ in } \textit{one} \text{ hour } \frac{360}{24} = 15^\circ \text{ pass the sun,}$$

i.e., of the 360° of longitude, 15° will correspond to one hour.

Again, the 360° correspond to 24 hours, therefore 1° will correspond to $\frac{24}{360}$ h.

$$= \frac{24 \times 60}{360} = 4 \text{ m.}$$

To convert arc into time: Multiply by 4 and divide by 60.

1. Convert 40° 17′ 18″ into time.

$$\begin{array}{r} 40^\circ\ 17'\ 18'' \\ 4 \\ \hline 6,0)\overline{16,1^\circ\ \ 9'\ 12''} \\ \hline 2\text{h. } 41\text{m. } 9{\cdot}2\text{sec.} \end{array}$$

2. Convert 130° 25′ 43″ into time.

$$\begin{array}{r} 130^\circ\ 25'\ 43'' \\ 4 \\ \hline 6,0)\overline{52,1^\circ\ 42'\ 52''} \\ \hline 8\text{h. } 41\text{m. } 42{\cdot}8 \text{ sec.} \end{array}$$

3. The difference of longitude between two places is 98° 41′ 36″ what it the difference in time? *Ans.* 6h. 34m. 46·4sec.

4. Convert 98° 27′ 19″ into time.

Ans. 4h. 33m. 49·2sec.

5. Given the difference of longitude between Plymouth and Bermuda 60° 30′ 24″; find the difference in time.

Ans. 4h. 2m. 1·6sec.

6. Plymouth is in longitude 4° 7′ 16″ W., and Iquique Island in 70° 13′ W.; find the difference in time.

Ans. 4h. 24m. 22·9sec.

One place is in longitude 125° 16′ 24″ E., another in 143° 18′ 36″ W; find the difference in time between the two places.

Ans. 6h. 5m. 40sec.

To convert time into arc: Bring the given time into minutes and seconds, and divide by 4. Work the decimals as in the second and third examples below.

7. Convert 14h. 10m. 45sec. into arc.

$$\begin{array}{r} \text{h. m. sec.} \\ 14\ \ 10\ \ 45 \\ 60\quad\quad \\ \hline 4)\overline{850\ \ 45} \\ \hline 212^\circ\ 41'\ 15''\ \textit{Ans.} \end{array}$$

8. The difference in time between Valparaiso and the Cape of Good Hope is 6h. 0m. 19·26sec.; what is the difference in longitude?

$$\begin{array}{r} \text{h.}\ \ \text{m.}\ \ \text{sec.} \\ 6\ \ \ 0\ \ 19{\cdot}26 \\ 60\quad\quad\quad \\ \hline 4)\overline{360\ \ 19\ \ 15{\cdot}6} \\ \hline 90^\circ\ 4'\ 48''{\cdot}9\ \textit{Ans.} \end{array} \qquad \begin{array}{r} {\cdot}26 \\ 60 \\ \hline 15{\cdot}60 \end{array}$$

9. What is the difference in longitude between two places when one is 4h. 10m. 12sec. east of Greenwich, and the other 5h. 7m. 3·5sec. west of Greenwich?

	h.	m.	sec.
Times	4	10	12
	5	7	3·5
	9	17	15·5
	60		
4)	557	15	30
	139°	18′	52·5 *Ans.*

10. When the clock shows 7h. 3m. 50sec. in the morning at New York, it is 2h. 2m. 53·46sec. in the afternoon at Odessa; find the difference of longitude.

	h.	m.	sec.	
	12	0	0	
New York	7	3	50	
	4	56	10	·46
Odessa	2	2	53·46	60
	6	59	3·46	27·60
	60			
4)	419	3	27·6	
	104°	45′	51″·9 *Ans.*	

11. Convert 138° 17′ 0″ into time.

Ans. 9h. 13m. 8sec.

12. The difference of longitude between two places is 73° 1′ 0″, what is the difference in time? *Ans.* 4h. 52m. 4 sec.

13. Find the difference of time between two places when the difference of longitude is 155° 32′ 10″.

Ans. 10h. 22m. 8·6sec.

14. When it is 2h. 10m. 15sec. in the afternoon at A it is six o'clock in the afternoon at B, what is the difference of longitude?

Ans. 57° 26′ 15″.

15. Convert 17° 18′ 41″·3 into time.

Ans. 1h. 9m. 14·75 sec.

16. It is required that 11h. 18m. 19sec. of time be converted into arc; find the arc. *Ans.* 169° 34′ 45″.

17. When it is 10h. 30m. A.M. at a place in west longitude it is 5h. 9m. 10sec. P.M. at one in east longitude, what number of degrees of longitude are the places apart? *Ans.* 99° 47′ 30″.

18. Convert 9h. 11m. 17·3sec. into arc.

Ans. 137° 49′ 19″·5

19. What number of degrees, minutes, and seconds of arc correspond with 15h. 11m. 18sec.? *Ans.* 227° 49′ 30″.

Before proceeding further with these questions, it will be well to give the reasons for the two foregoing rules, and then show clearly how to find the corresponding time at a place when we know the hour at another with the longitude of both places, or at least the difference of longitude.

It has been proved that 15° of longitude corresponded to one hour of time, therefore if we divide any given arc by 15, it must give us the corresponding time in hours, minutes, and seconds: since $15 = \frac{60}{4}$, it is evident that if we have to divide any given number by 15 it will come to the same thing if we multiply by 4 and divide by 60. It will be seen by a glance at the few preceding examples that it is much easier to multiply by 4 and divide by 60 than divide by 15. Hence the reason for the rule by which arc is converted into time.

The second rule, or the rule by which time is converted into arc, is derived also from Art. 58, where it was shown that 4m. of time are equivalent to 1° of arc. Hence it is evident that if we bring any given time into minutes, and divide by 4, we shall have the corresponding arc in degrees, minutes, and seconds of arc.

As regards the time at place when one place is east and the other west of Greenwich, this will easily be understood if we suppose that as the earth turns round from west to east, the places to the east will come opposite the sun before those lying farther west. Bombay will turn toward the sun before Aden, Aden before Constantinople, Constantinople before Paris, Paris before New York, etc. Hence it is evident that if one place is east of another, at the place farther east it is *later* in the day; while if another be to the west, there it is earlier in the day. Bear this well in mind, and no difficulty will arise as to which way we must apply arc or time to obtain the hour of the day at any given place.

20. It is six o'clock in the morning at Constantinople in longitude 28° 59′ 2″ E., what time is it at Cape Farewell, latitude 59° 49′ 12″ N., longitude 43° 53′ 40″ W.?

The total difference of longitude is { 28° 59′ 2″ E.
43° 53′ 40″ W

72° 52′ 42″
4

6,0)29,1° 30′ 48″

4h. 51m. 30·8sec.

Time at Constantinople, 6h. 0m. 0sec.

At Cape Farewell it is, 1h. 8m. 29·2sec.

Ans. 1h. 8m. 29·2sec. in the *morning.*

Astronomically it is 18h. 0m. 0sec. at Constantinople, while it is 13h. 8m. 29·2sec. at Cape Farewell same day.

21. With the above question find what time it is at Bombay, 72° 54′ 24″ E. longitude.

Difference of longitude { 28° 59′ 2″ E.
72° 54′ 24″ E.

43° 55′ 22″
4

6,0)17,5° 41′ 28″

2h. 55m. 41·4sec.

Time at Constantinople, 6h. 0m. 0sec.

Time at Bombay......... 8h. 55m. 41·4sec. in the morning.

Astronomically it is 20h. 55m. 41·4sec. at Bombay same day.

Thus we see that when it is one o'clock in the morning at Cape Farewell, it is six in the morning at Constantinople, and nearly nine at Bombay; while at Aurora Island, as shown by the next question, it is about three o'clock in the afternoon of next day.

22. It is 8h. 55m. 41·4sec. in the morning at Bombay in longitude 72° 54′ 24″ E., what time is it at Aurora Island in the New Hebrides, latitude 14° 53′ S., longitude 168° 16′ E.?

Difference of longitude, { 72° 54′ 24″ E.
168° 16′ 0″ E.

95° 21′ 36″
4

6,0)38,1° 26′ 24″

6h. 21m. 26·4 sec.

	h. m. sec.
Time at Bombay.........	8 55 41·4
Difference in time.......	6 21 26·4
Time at Aurora Island	15 17 7·8, or 3h. 17m. 7·8sec. in the afternoon of next astronomical day.

To find the Greenwich date from the chronometer.

Through the ambiguity of the chronometer, arising from the fact that it only shows 12 hours of time instead of 24, there is sometimes a momentary difficulty when at sea in telling what the time is; for instance, if we see a watch showing ten o'clock, the question arises is it ten in the morning or ten at night? The difficulty is overcome by roughly applying the ship's longitude in time. We have hence the following rule:—

If it be P.M. the watch shows the correct Greenwich date; if A.M. add twelve hours, and call it the day before.

This point is more fully explained presently.

23. It is May 12 day 2h. at ship in longitude 17° 25′ 30″ E., the chronometer shows 50 minutes past 12, what is the Greenwich date?

	h. m. sec.
Time at ship, 12 day,..........	2 0 0
Longitude E....................	1 9 43·3
Greenwich date, May 12 day	0 50 16·7

Longitude in time.
17° 25′ 50″ E.
4
6,0)6,9° 43′ 20″
1h. 9m. 43·3sec.

24. The chronometer shows 9h. 20m. 10sec. A.M., what is the time at ship P.M., May 12 day in longitude 120° 13′ 14″ E.?

	h. m. sec.
Greenwich date, May 11 day	21 20 10
Longitude in time	8 0 52·9
Time at ship May 12 day....	5 21 2·9

Longitude in time.
120° 13′ 14″ E.
4
6,0)48,0° 52′ 56″
8h. 0m. 52·9sec.

25. The chronometer shows 9h. 20m. 10sec. A.M., what is the correct time at ship A.M. May 12 day in longitude 120° 13′ 14″ W.?

	h. m. sec.
Greenwich date, May 11 day	21 20 10
Longitude in time W.	8 0 52·9
Time at ship May 11 day ...	13 19 17·1

Longitude in time.
120° 13′ 14″ W.
4
6,0)48,0° 52′ 56″
8h. 0m. 52·9sec.

26. September 4 day 1874, at 5h. 49m. P.M. mean time nearly at ship, in longitude by account 147° 18′ W., a chronometer showed 3h. 35m. 18sec., and its error on Greenwich mean time was 4m. 18sec. slow, what is the Greenwich date?

Greenwich date from watch.		Longitude in time.
Sept. 4 day	3h. 35m. 18 ec.	147° 18′ W.
Error	+4m. 18sec.	4
	3h. 39m. 36sec.	6,0)58,9° 12′
		9h. 49m. 12sec.

Greenwich date nearly.
Sept. 4 day 5h. 49m. 0sec.
9h. 49m. 12sec.
Sept. 4 day 15h. 38m. 12sec.

∴ G. D. is 15h. 39m. 36sec.

i.e., the 3 hours shown is 3 o'clock in the morning at Greenwich.

27. 1874, May 19 day, 7h. 3m. A.M. mean time at ship nearly, in longitude by account 56° 49′ W., a chronometer, which was 12m. 4sec. slow on Greenwich mean time, showed 10h. 38m. 2sec., required the Greenwich date.

Greenwich date.		Longitude in time.
Chron. shows	10h. 38m. 2sec.	56° 49′ W
Slow	+12m. 4sec.	4
	10h. 50m. 6sec.	6,0)22,7° 16′
		3h. 47m. 16sec.

Greenwich date nearly.
May 18 day 19h. 3m. 0sec.
Lon. W. ... 3h. 47m. 16sec.
22h. 50m. 16sec.

∴ as it is A.M. at Greenwich, the correct astronomical date is May 18 day 22h. 50m. 6sec.

Hence we obtain the following rules:—

To find the Greenwich date:

(1) Turn the longitude into time.

(2) Apply it to the ship's date, so as to find the Greenwich date nearly.

(3) To the Greenwich date apply the error and rate, then by looking at the date as found from the ship's reckoning in (2), we see whether it is A.M. or P.M. If it is P.M., the day at ship is the Greenwich date; if it is A.M., call it the day before, and add 12 hours.

28. 1874, June 10 day, 10h. 18m. 4sec. (nearly) A.M. at ship, in longitude 47° 18′ 15″ E., the chronometer, which was fast 10m. 14sec. on April 15 day, and gaining daily 2·5 seconds, showed 7h. 21m. 20sec., what is the correct date at Greenwich?

Greenwich date.	h. m. sec.
Chronometer shows	7 21 20
Fast..................	0 10 14
	7 11 6
Rate.................	−2 17·5
	7 8 48·5

Greenwich date nearly.	h. m. sec.
June 10 day	10 18 4
Lon. E.......	3 9 13
June 10 day	7 8 51 A.M.

Longitude in time.

47° 18′ 15″ E.
4
6,0)18,9° 13′ 0″
3h. 9m. 13sec.

Rate.

2·5
55
125
125
6,0)13,7·5
2m. 17·5sec.

As the date is A.M., we add 12 hours and call it
June 9 day 19h. 8m. 48·5 sec. Greenwich date.

The rate is found by taking the number of days elapsed since the error was found, and multiplying by it, which will certainly give the whole gain or loss for the time.

29. It is 5h. 30m. in the afternoon in longitude 21° 16′ 17″ E., what time is it at Greenwich, and what time in longitude 21° 16′ 17″ W.?

Ans. W. lon. 2h. 39m. 49·8sec.; Greenwich date 4h. 4m. 54·9sec.

30. Find what time it is in longitude 121° 17′ 16″ E. and 80° 19′ 20″ W., when it is May 2 day 3h. 20m. at Greenwich.

Ans. May 2 day 11h. 25m. 9sec.
May 1 day 21h. 58m. 42·7sec.

31. Find what time it is in longitude 117° 18′ 21″ W., when it is May 3 day 7h. 18m. 21sec. at Greenwich.

Ans. May 2 day 23h. 29m. 7·6 sec.

32. Find what time it is in longitude 99° 18′ 17″ E., when it is June 4 day 18h. 12m. in longitude 21° 10′ 18″ W.

Ans. June 5 day 2h. 13m. 54·3sec.

33. At ship it is October 17 day 10h. 12m. A.M. nearly in longitude 65° 14′ 18″ E. by account; what is the correct Greenwich date when the watch, which is 25m. 19sec. too fast, shows 5h. 51m. 4sec.?

Ans. Oct. 16 day 17h. 25m. 45sec.

34. December 14 day 8h. 12m. P.M. at ship nearly in longitude 104° 16′ E., a chronometer which was 15m. 12sec. too slow, on August 10 day, and losing daily 1·5 seconds, showed 0h. 56m. 20sec., what is the Greenwich date?

Ans. December 14 day 1h. 14m. 41sec.

EXAMPLES AND QUESTIONS TAKEN FROM THE GOVERNMENT EXAMINATION PAPER.

1. Give definitions of the following terms: circles of declination, circles of latitude, right ascension of a heavenly body, declination of a heavenly body (1863).

2. Give definitions of the following terms: a great circle, meridian, prime vertical, ecliptic, azimuth, and longitude of a heavenly body (1863).

3. Define the following terms: (1) meridian; (2) zenith; (3) nadir; (4) altitude; (5) azimuth; (6) latitude and longitude of a star; (7) hour angle (1863, 1864).

4. If P be the pole of the heavens, Z the zenith of a place on the earth's surface, S the place of a heavenly body; what will the sides and angles of the spherical triangle P Z S respectively represent (1863, 1864)?

5. Explain what is meant by sidereal time, apparent solar time, mean solar time, and equation of time. Given apparent time, 4h. 30m. P.M., and clock before the sun 3m. 0sec.; what is the mean time (1864)? *Ans.* 4h. 33m.

6. Define altitude, azimuth, right ascension, and declination (1866). Define the ecliptic and obliquity of the ecliptic, and latitude and longitude of a heavenly body (1867).

7. Define right ascension and declination of a heavenly body, also altitude and zenith distance (1868).

8. Define amplitude and azimuth of a heavenly body. Illustrate by a diagram (1868).

9. Define ecliptic, horizon, azimuth, right ascension, latitude of a heavenly body. Show how the position of a heavenly body may be determined by referring it, with the aid of another great circle, (1) to the equinoctial, and (2) to the horizon (1869).

10. Give definitions of ecliptic, celestial meridian, declination of a heavenly body, amplitude of a heavenly body, mean sun. Explain your definitions by figures (1871).

11. Give, without reference to figures, definitions of the following: ecliptic, horizon, right ascension, declination, hour angle. Explain these definitions by the aid of diagrams (1870).

12. Define right ascension and declination of a heavenly body; also altitude and zenith distance (1867).

CHAPTER III.

DAYS, YEARS, HOUR ANGLES, AND MERIDIAN PASSAGES.

Definitions—Lengths of Sidereal and Solar Days—Lengths of Mean and Sidereal Year — Right Ascension, to Find and Correct for a Place East or West of Greenwich—Hour Angle —Graphic Mode of Showing Mean Apparent and Solar Time with Equation of Time—To Convert Sidereal to Mean Time and Mean to Sidereal Time—To Find Right Ascension of Meridian—To Find the Time a Star will Pass the Meridian —To Find what Stars are near the Meridian—To Find Hour Angles—To Find Sidereal Time—Examination.

In commencing this chapter, a few definitions are first repeated.

In Astronomy there is the solar day, mean solar day, the sidereal day, and a lunar day.

59. A Solar Day is the time between the sun passing the meridian of a place till it returns to it again. They vary in length. Solar or *true* time is measured by the interval between two noons, or actual passage of the meridian of any place over the sun; and it follows, because the time is not the same at all seasons of the year, that solar time varies, and that a solar hour, the twenty-fourth part of a solar day, is not of the same length on two successive days.

60. A Mean Solar Day is a day as shown by the mean sun, or the interval that elapses between two successive returns of the mean sun to the same meridian.

61. A Sidereal Day is the time of one absolute revolution of the earth, or the time that elapses between two transits of the same star, or it is the time between

two successive returns of the meridian to the same star.

62. A Lunar Day is the interval that elapses between two returns of the moon to the same meridian.

63. Mean Solar Time is the time shown by the mean sun, or by a good clock or watch.

64. Apparent Time is the time shown by the sun, or by a good sun-dial.

A sidereal day may be said to begin and end when the first point of Aries crosses the meridian of Greenwich. If it should happen to be the vernal equinox when the first point of Aries is on the meridian, then the sun will be there at the same time. Now, if both move off together, it will be found that the first point of Aries will get to the meridian again before the sun does; or at the end of a sidereal day. Because, as the earth turns round, she moves forward in her orbit, and, as it were, passes the sun; this has to be made up, or the earth has to turn a little more than quite round for the Greenwich meridian to face the sun again. Looking closely at the above fact, we shall see the reason why there is one more sidereal day in a year than there are solar days, or a sidereal day is $\frac{1}{366}$ shorter than a solar (nearly). This is the reason why a seaman sailing east round the world gains a day, while if he sails west he loses a day.

A sidereal day = ·9973 mean solar days.
A solar day = 1·002738 sidereal days.

The sidereal day begins at *12 o'clock* on the 21st of March and 23rd of September, but never commences at the same time as a mean solar day at any other period.*

65. Length of the Day.—A mean solar day is twenty-four hours of mean solar time, or 24h. 3m. 56·5584sec. of sidereal time.

A sidereal day is 23h. 56m. 4·0906sec. of mean solar time; of course, it consists of 24 hours of sidereal

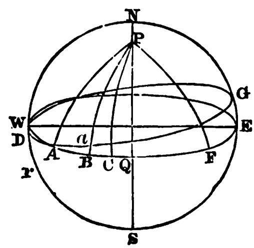
* See Denison's *Astronomy Without Mathematics.*

time. Much learning, deep research, and minute calculation, have been required to find the exact length of the day and year.

66. The Length of the Mean Solar or Tropical Year, as determined by our great astronomers, consists of 365d. 5h. 48m. 51·6sec.

67. The Length of the Sidereal Year has been determined to consist of 365d. 6h. 9m. 11·5sec.

68. Equation of Time is the difference between mean and apparent time. It is the interval that the clock is before or after the sun.



69. Right Ascension (R.A.) of the Mean Sun is its distance from the first point of Aries along the celestial equator.

70. Right Ascension (R.A.) of the Meridian is its distance from the first point of Aries along the celestial equator.

Suppose this figure to represent the celestial sphere.

Let W Q E be the equator.

Let D A G be the ecliptic.

Let N S be the meridian of Greenwich.

Draw P A, P B, P C, and P F, parts or quadrants of circles of declination.

Let us consider A as the first point of Aries, and *a* where the declination circle PB cuts the ecliptic as the position of the sun in the ecliptic, and C the place of the mean sun in the equator.

We are now in a position to show clearly by the figure what is meant by right ascension (R.A.).

The R.A.	of	the meridian	is AQ or APQ.
,, R.A.	,,	mean sun	is AC or APC.
,, R.A.	,,	true sun	is AB or APB.

If the mean sun be at C, then the angle CPQ is

measure of mean time; if at F, the angle FPQ is the measure of mean time *before* noon.

If the apparent sun be at *a*, the angle BPQ is the measure of apparent time, therefore the angle CPB is the measure of the equation of time.

While APQ, the R.A. of the meridian, is the measure of sidereal time.

$$\therefore \text{ R.A. of meridian} = \angle APC + \angle CPQ$$
$$= \text{R.A. of mean sun} + \text{mean time}$$

Also if we suppose C to be the place of a star (or it may be any where on the line PC).

$$\therefore \begin{matrix}\text{R.A. of meridian} \\ \text{or sidereal time}\end{matrix} = \angle\ APC + CPQ$$
$$= \text{R.A. of star} + \text{its mer. dis. (west).}$$

Again $QPC = QPA - APC$
$= \text{R.A. of meridian} - \text{R.A. of star}$;
but R.A. of meridian = R.A. of mean sun + mean time.

$\therefore$ QPC or Hour angle of star = R.A. of mean sun + mean time − R.A. of star.

Or mean time = Hour angle of star + R.A. of star − R.A. of mean sun.

These two last equations are of the utmost importance in nautical astronomy.

We now proceed to show the utility of all of these equations, but must first show the use of the tables of time equivalents in the *Nautical Almanac*, pp. 522 to 525.

1. Convert 16h. 17m. 18sec. of mean time into sidereal time. From page 522.

Mean.	Sidereal. h.	m.	sec.	
16h. =	16	2	37·7036	
17m. =		17	2·7927	
18sec. =			18·0493	
	16	19	58·5456	*Ans.*

Go to the *Nautical Almanac*, page 522, in the first column, opposite 16 hours is seen 16h. 2m. 37·7036sec. This is set down first. Next, in the second division of the page, we have the time equivalents for minutes, opposite 17 minutes is seen 17m. 2·7927sec.; this is put

under the first quantity. In the third division of the page is found the time equivalents for seconds; opposite 18 seconds is found 18·0493sec., which we place under the other two, and adding up the whole gives a result of 16h. 19m. 58·5456sec. of sidereal time as the equivalent of 16h. 17m. 18sec. of mean solar time.

Otherwise the student must remember that 24 hours mean time = 24h. 3m. 56·5554sec. of sidereal time, and state the sum thus :—

	h.		h.	m.	sec.		h.	m.	sec.	
As	24	:	16	17	18	: :	24	3	56·5554	: Ans.

This is a very tedious process compared with the former.

2. It is required to convert 16h. 17m. 18sec. of sidereal time into mean time.

Sidereal.	Mean. h. m. sec.	
16h. =	15 57 22·7270	
17m. =	16 57·2150	
18sec. =	17·9509	
	16 14 37·8929	*Ans.*

Turn to page 524 and 5 of the *Nautical Almanac*, and proceed precisely as in the last example. It will of course be perceived that each quantity is less than the one sought, while in the previous case each was greater.

3. What is the equivalent in mean time of 20h. 9m. 12·5sec. of sidereal time? *Ans.* 20h. 5m. 54·4003sec.

4. Reduce 24h. 50m. 19·8sec. of sidereal mean time. *Ans.* 24h. 46m. 15·6452sec.

5. Express 5 days 10h. 18m. 12sec. of mean time in sidereal time. *Ans.* 130h. 39m. 36·3315 sec.

6. Express 5 days 10h. 18m. 12sec. of sidereal time in mean time. *Ans.* 5 days 9h. 56m. 51·1757 sec.

7. Express 9h. 17m. 11·99sec. of mean time as sidereal time. *Ans.* 9h. 18m. 43·5238 sec.

8. What is the interval of mean time equivalent to 11h. 39m. 56·4sec. of sidereal time? *Ans.* 11h. 38m. 1·7316sec.

To find the R.A. of the meridian.

R.A. of meridian = mean time + R.A. of mean sun.

9. Find the R.A. of the meridian for November, 21d. 10h. 18m. 12sec.

		h.	m.	sec.
Sidereal time at mean noon Nov. 21d. =		16	1	16·16
Converting the mean into sidereal time,	10h. =	10	1	38·5647
	18m. =		18	2·9569
	12sec. =			12·0329
R.A. of meridian is		26	21	9·7145

10. Had we converted the mean into sidereal time, we should have found the R.A. of mean sun; that is found more exactly thus :—

		h.	m.	sec.
Sidereal time at mean noon		16	1	16·16
Acceleration for	10h.		1	38·5647
	18m.			2·9569
	12sec.			·0329
R.A. of mean sun,		16	2	57·7145

11. The R.A. of the mean sun and the R.A. of the meridian are required for 1874, May 15d. 19h. 11m. 44sec.

		h.	m.	sec.
Sidereal time at mean noon May 15, is		3	32	10·48
Acceleration for	19h.		3	7·2730
	11m.			1·8070
	44sec.			·1205
R.A. of mean sun is		3	35	19·6805
Mean time at place,		19	11	44·
R.A. of meridian,		22	47	3·6805

Hence, to find the R.A. of the meridian, we must first take the sidereal time from the *Nautical Almanac*, then add the acceleration for the mean time by taking it from the table of time equivalents. This acceleration, added to the sidereal time, gives the R.A. of the mean sun; to obtain the R.A. of the meridian from this, we add the mean time at place, omit 24 if it be over 24 hours.

12. Find the R.A. of the meridian and R.A. of the mean sun for 1874, February 10d. 18h. 9m. 11sec. Sidereal time, February 10=21h. 21m. 34·35sec.

Ans. R.A. mean sun 21h. 24m. 33·2751sec.
R.A. meridian 15h. 33m. 44·2751sec.

13. Find the R.A. of the meridian and R.A. of the mean sun for 1874, June 21d. 9h. 8m. 12sec. Sidereal time, June 21=5h. 58m. 3·13sec.

Ans. R.A. mean sun 5h. 59m. 33·1854sec.
R.A. meridian 15h. 7m. 45·1854sec.

To find what time a star will pass the meridian.—When a star passes the meridian, or is on the meridian, its hour angle, or distance from the meridian, vanishes; hence the equation before proved, viz.,

Hour angle = mean time + mean sun's R.A. − R.A. of star
becomes 0 = mean time + R.A. of mean sun − R.A. of star.
∴ Mean time = R.A. of star − R.A. of mean sun.

14. Find at what time α Aquilæ (Altair) will pass the meridian of New York, lon. 74° 2′ 30″ W., on the 28th May, 1874.

	h.	m.	sec.	
R.A. of star Altair	19	44	39·69	
R.A. of mean sun	4	23	25·73	
Ans. May 28d.	15	21	13·96	nearly.

This gives the answer correctly for most purposes for which the passage of a star is wanted; but when required with greater accuracy, the process must be continued thus—

Greenwich Date.		h. m. sec.	Longitude in Time.
Date at New York nearly,	May 28d.	15 21 13·96	74° 2′ 30″ W.
			4
	Lon. W.	4 56 10	6,0)29,6° 10′ 0″
	May 28d.	20 17 23·96	4h. 56m. 10sec.

R.A. of Mean Sun.

		h. m. sec.
Sidereal time		4 23 25·73
Accel. for	20h.	3 17·1295
	17m.	2·7927
	23sec.	·0603
		4 26 45·7125

Transit of Star.

	h. m. sec.
R.A. of Altair	19 44 39·69
R.A. of meridian	4 26 45·71
May 28d	15 17 53·98

Ans. Time of transit or meridian passage, May 28d. 15h. 17m. 53·98sec.

15. Find the time when α Scorpii (Antares) will pass the meridian of Cape Guardafui, longitude 51° 16′ E. on March 6d., 1874.

	h.	m.	sec.	
R.A. of Antares	16	21	40·97	
R.A. of mean sun	22	56	11·66	
Transit, March 6d.	17	25	29·31	nearly.

Greenwich Date.		h.	m.	sec.	Longitude in Time.
M.T. at Cape Guar$^{d.}$	March 6d.	17	25	29·31	51° 16′ 0″ E.
					4
Longitude in time E.		3	25	4	6,0)20,5° 4′ 0″
	March 6d.	14	0	25·31	3h. 25m. 4sec.

R.A. of Mean Sun.

	h.	m.	sec.
Sidereal time	22	56	11·66
Accel. for 14h.		2	17·9906
Accel. for 25 sec.			·0685
	22	58	29·7191

Transit of Star.

	h.	m.	sec.
R.A. of Antares	16	21	40·97
R.A. of mean sun	22	58	29·71
	17	23	11·26

∴ Time of transit or meridian passage, is March 6d. 17h. 23m. 11·26sec.

Hence we have the following rule for finding what time a given star will be on the meridian of the observer :—

(*a*) Subtract the R.A. of the mean sun from the R.A. of the star. This gives the mean time nearly from the formula : *Mean time* = R.A. of star − R.A. of mean sun.

(*b*) With this mean time of place and longitude find the Greenwich date.

With this date find the correct R.A. of the mean sun or sidereal time, and subtract it from the R.A. of the star : this gives the time of the meridian passage to within a second of time or so.

16. Find at what time α Hercules will pass the meridian of a place in longitude 98° 17′ W. on April 10d., 1874.

	h.	m.	sec.
Sidereal time =	1	14	11·04
R.A of α Hercules =	17	8	55·13

Ans. Ap. 10d. 15h. 51m. 2·67sec.

17. Can you state exactly the time at which α Cygni will be on the meridian of a place in longitude 125° W. on January 1d., 1874.

	h.	m.	sec.
Sidereal time	18	43	52.56
R.A. of α Cygni	20	37	5·81

Ans. January 1d. 1h. 51m. 23·01sec.

18. Give the time of the meridian passage of Arcturus in longitude 4° 16′ W. on January 31d., 1874.

	h.	m.	sec.
Sidereal time	20	42	8·79
R.A. of Arcturus	14	9	54·91

Ans. January 31d. 17h. 24m. 51·2sec.

19. On 1874, December 9d. find the time of transit of β Aquilæ in longitude 45° 16′ 20″ E.

	h.	m.	sec.	
Sidereal time	17	8	17·69	(Dec. 8d.
R.A. of β Aquillæ	19	49	8·66	

Ans. 2h. 36m. 57·74sec.

To find what stars are near any meridian:—

It is evident that if a star is near the meridian of any place, its right ascension must be equal to the right ascension of the meridian. Hence if we find the R.A. of the meridian of a place, and then look for the bright stars in the *Nautical Almanac* whose right ascensions most nearly coincide with that of the meridian, those are the stars required.

As was previously stated, when a star is on the meridian its hour angle vanishes.

∵ *hour angle* = mean time + R.A. mean sun − R.A. of star
∴ 0 = mean time + mean sun's R.A. − star's R.A.
∴ star's R.A. = mean time + mean sun's R.A.
= R.A. of the meridian.

Therefore, to find the stars near the meridian we must

(*a*) Find the Greenwich date;

(*b*) Find the R.A. of the mean sun;

(*c*) Add the mean time at place to the R.A. of mean sun, which gives the R.A. of the meridian.

(*d*) Knowing now the R.A. of the meridian, we must go to the *Nautical Almanac*, and look out the stars whose right ascensions coincide with it.

20. Aug. 20 day, 1874, at half-past 9 in the evening, what bright stars are near the meridian in lon. 65° 17′ 20″ W.?

Greenwich Date.

	h.	m.	sec.
Time at place, Aug. 20d.,...	9	30	0
	4	21	9·3
Aug. 20d.,..	13	51	9·3

Longitude in Time.

65° 17′ 20″ W.
4
6,0)26,1 9 20
4h. 21m. 9·3sec.

R.A. of Mean Sun.				R.A. of Meridian and Star.			
	h.	m.	secs.		h.	m.	secs.
Sidereal time =	9	54	36·57	R.A. mean sun =	9	56	53·1068
Accel. 13h. =		2	8·1342	M. time at place =	9	30	0
Accel. 51m. =			8·3780		19	26	53·1068
Accel. 9sec. =			·0246				
	9	56	53·1068				

Going to the *Nautical Almanac*, and looking in the catalogue of bright stars whose R.A. and declinations are given, it is found that the following stars are near the meridian of the observer in lon. 65° 17′ 20″ at half-past nine in the evening :—

ω, δ, and γ Aquilæ and h^2 Sagittarii.

The constellation of the Eagle is exactly south from the spectator.

21. Find what bright stars are near the meridian of a place in lon. 124° 16′ 10″ E. on June 21 day 9h. 18m. 12sec. mean time at place.

Greenwich Date.	h.	m.	sec.	Longitude in Time.
				124° 16′ 10″ E.
M. T. at place, June 21d.	9	18	12	4
	8	17	4·6 E.	6,0)49,7 4 40
M. T. at Greenwich, 21d.	1	1	7·4	8h. 17m. 4·6sec.

R.A. of Mean Sun.	h.	m.	sec.	R.A. of Meridian and Stars.	h.	m.	sec.
Sidereal time =	5	58	3·13	R.A. of mean sun ..	5	58	13·17
Accel. 1h. =			9·8565	Mean time	9	18	12
Accel. 1m. =			·1643		15	16	25·17
Accel. 7sec. =			·0192				
	5	58	13·1700				

The stars having their R.A. nearly equal to 15h. 16m. 25·57sec. are β Libræ and α Coronæ.

22. Find what bright stars are near the meridian of St. Helena, lon. 5° 44′ W., at 10 o'clock in the evening of October 1, 1874.

Fomalhaut, γ Piscium, and α Pegasi.

To find the hour angle of a star.—The hour angle ... elestial object is its distance westwards from the ... of Greenwich. The hour angle of the sun is

the apparent time at place: of other celestial objects we have just proved that

Hour angle = mean time + R.A. of mean sun − R.A. of star

23. Find the hour angle of Vega when the Greenwich mean time is 9h. 10m. 18sec. P.M., Sept. 18d.

R.A. of Mean Sun.

		h.	m.	sec.
Sidereal time, *N. A.*, II. page,		11	48	56·64
Accel. for	9h.		1	28·7083
	10m.			1·6428
	18sec.			·0493
		11	50	27·0404

Hour Angle.

	h.	m.	sec.
Mean time at Greenwich,	9	10	18
R. A. of Mean Sun,.......	11	50	27·0404
	21	0	45·0404
R. A. of Vega,.............	18	32	41·77
	2	28	3·2704
	60		
4)	148	3	16·2
Hour angle,	37°	0′	49″ W.

24. Required the hour angle of the Pole Star on September, 18d., at 10h. 11m. 25sec. mean time, in longitude 12° 10′ 18″ W.

Greenwich Date.

	h.	m.	sec.
M. T. at place, Sept. 18d.	10	11	25
Longitude in time, W.......	0	48	41·2
G. M. T., ... Sept. 18d.	11	0	6·2

Lon. in Time.

	12°	10′	18″ W.
			4
6,0)	4,8	41	12
	0	48m.	41·2sec.

R. A. of the Mean Sun.

			h.	m.	sec.
Sidereal time from *N. A.*		=	11	48	56·64
Accel. for	11h.	=		1	48·4212
	6sec.	=			·0164
			11	50	45·0776

Hour Angle.

	h.	m.	sec.
Mean time at place, ...	10	11	25
R. A. of Mean Sun, ...	11	50	45·0776
	22	2	10·0776
R. A. of Polaris, ...	1	13	15·63
Hour angle W. in time,	20	48	54·4476
	24		
Hour angle E. in time,	3	11	5·5524
	60		
4)	191	5	33·1
Hour angle E. in arc,	47°	46′	23″·2

Rule for finding Hour Angles:—

(*a*) Find the mean time at Greenwich or Greenwich date.

(*b*) Take out the sidereal time, and find the R. A. of the mean sun for that date.

(*c*) To the mean time at place, add the R. A. of the mean sun; from this subtract the R. A. of the object: this gives the hour angle in time.

(*d*) Turn the hour angle into arc in the usual manner, by bringing the hours into minutes, and dividing by 4.

(*e*) If the hour angle is greater than 12 hours, subtract from 24 hours, and call it E., otherwise it is W.

(*f*) The hour angle marked E. or W. simply means this: If W. it is west of the meridian, if E. it is east of the meridian. The angles properly should all be westerly, but, as it is often more convenient to have them under 180°, we subtract when above 12 hours from 24 hours, and call the angle E.

25. Find the hour angle of Spica, 1874, Oct. 8d. 12h. 19m. 10sec. mean time at place in longitude 49° 10′ W.

R. A. of Spica is 13 18 33·92
Sidereal time, 13 7 47·72

Ans. 177° 15′ 36″·99 E.

26. Required the hour angle of β Leonis, 1874, Oct. 25d. 9h. 7m. 11sec. mean time at place in longitude 84° 16′ E.

R. A. of β Leonis, 11 42 39·05
Sidereal time,...... 14 14 49·14

Ans. 174° 58′ 54″·1 W.

27. What is the hour angle of the Pole Star, Oct. 8d. 17h. 11m. mean time at place in longitude 120° E.?
R. A. of Pole Star, 1h. 13m. 21sec. } *Ans.* 76° 44′ 18″·5 W.

71. Sidereal Time.—We have omitted to make clear what is meant by Sidereal Time, as taken from the second page of the month of the *Nautical Almanac.* The sidereal time there given is the sidereal time at mean noon at Greenwich, which also corresponds with the right ascension of the mean sun. Suppose the equation, as proved in a preceding page, be taken.

Sidereal time = mean time + R.A. of mean sun,

Now at mean noon mean time vanishes, so then we have

Sidereal time = R.A. of mean sun,

And this is exactly what is given in the *Nautical Almanac*, and is what a clock would show that keeps sidereal time; it is, in fact, the angular distance of the first point of Aries from the meridian at the instant the centre of the mean sun is on the meridian. It expresses the actual hour angle from the meridian westward of the true (vernal) equinoctial point at the moment of mean noon at Greenwich.

28. The mean time at Sydney in longitude 151° 14′ E. is May 18d. 7 h. 10m. 19sec : find the sidereal time

Sidereal time = mean time + R.A. of mean sun.

Greenwich Date.		h.	m.	sec.		Longitude in Time.
M. T. at Sydney,	May 18d.	7	10	19		151° 14′ E.
Long. in time,		10	4	56	E.	4
	May 17d.	21	5	23		6,0)60,4 56
						10h. 4 m. 56sec.

R.A. of Mean Sun.

		h.	m.	sec.
R.A. of meridian or sidereal time =		3	40	3·59
	21h.		3	26·9859
	5m.			0·8214
	23sec.			·0630
Sidereal time - - -		3	43	31·4603
Mean time at Sydney - -		7	10	19
Sidereal time at ,, - -		10	53	50·4603

29. The mean time at the Island of Jan Mayen, in longitude 7° 26′ W., is July 21d. 21h. 21m. 21sec: find the sidereal time.

Greenwich Date.	h. m. sec.	Longitude in Time.
M. T. at Jan Mayen, July 21d.	21 21 21	7° 26′ W.
Longitude in time W.	29 44	4
G. M. T. July 21d.	21 51 5	6,0)2,9 44
		29m. 44sec.

R.A. of Mean Sun.

Sidereal time, July 21d.	=	7 56 19·88
Accel. for { 21h.	=	3 26·9859
51m.	=	8·3780
5sec.	=	·0137
Sidereal time at Jan Mayen		7 59 55·2576
		21 21 21
Sidereal time at place.........		5 21 16·2576

Rules for finding Sidereal Time at any place:—

(1) Find the Greenwich date.

(2) Find the R.A. of the mean sun.

(3) Find the sum of the mean time at place, and R.A. of the mean sun; this, omitting 24 hours when over that number, is the required sidereal time at the place given.

30. Find the sidereal time at Teneriffe, longitude 16° 39′ W., on the 10th July, 1874, at 9h. 15m. 10sec. P.M. mean time at place. Sidereal time = 7h. 12m. 57·74sec.

Ans. 16h. 29m. 49·8804sec.

31. Required the sidereal time at a place in longitude 157° 16′ 20″ E. on March, 17d. 19h. 12m. 14sec. mean time at place. Sidereal time = 23h. 39m. 33·75.sec.

Ans. 18h. 53m. 13·6902sec.

32. What is the sidereal time at Plymouth, longitude 4° 12′ W., on the 14th February, 1854, at 9h. 16m. 13sec. A.M.

Sidereal time, February, 13d. = 21h. 33m. 24·02sec.
14d. = 21h. 37m. 20·57sec.

Ans. 18h. 53m. 9·4297sec.

33. Give the sidereal time in longitude 120° 10′ E., when the time shown by a chronometer, correct for Greenwich mean time, is March, 12d. 9h. 12m. 14sec.

Sidereal time 23h. 19m. 50·98sec. *Ans.* 16h. 34m. 15·6979sec.

34. How is the length of a mean solar year determined? Why is a mean solar year shorter than a sidereal year, and a mean solar day longer than a sidereal day (1863)?

35. Show by means of a diagram what is meant by sidereal

time, apparent time, mean solar time, and the equation of time (1863).

36. What is meant by mean time and sidereal time, and show how one may be converted into the other?

37. What is the sidereal time of a place whose longitude is 125° W. of Greenwich on June 11, 1874, at 3h. 40m. local mean time (1863)? Sidereal time 11d. = 5h. 18m. 37·54sec.

Ans. 9h. 0m. 35·8177sec.

38. Having given the sidereal time at any instant, show how to find mean time at the same instant?

39. Having given the sidereal time = 12h. 10m. 10sec., and the right ascension of the mean sun at mean noon at the place = 1h. 42m. 14·5sec., required the correct mean time (1864).

Ans. 10h. 27m. 55·5sec.

40. If 24 mean solar hours = 24h. 3m. 56.555sec. sidereal hours, express 3 days 8 hours mean solar time in sidereal time (1864). *Ans.* 3d. 8h. 13m. 8·518sec.

41. The length of the mean solar year being 365·242264 days, find the daily motion of the mean sun (1865). *Ans.* 59′ 8″19·7.

42. In a mean solar day a meridian of the earth revolves through 360° 59′ 8″·33. Explain this (1865).

43. Explain accurately what is meant by mean, apparent, and sidereal time, illustrating by a figure (1866).

44. What is meant by equation of time? Show accurately the causes on which it depends. Show that the equation of time vanishes four times a year (1866).

45. What is meant by sidereal time? and what by mean solar time? Convert 15h. 12m. 18sec. sidereal time into mean solar time (1866). *Ans.* 15h. 9m. 48·5416sec.

46. Why is a sidereal year longer than a mean solar year (1866)?

47. Define a sidereal day, sidereal time, mean solar day, mean solar time. What is the sidereal time given on page II. of the *Nautical Almanac?*

48. Given that a meridian of the earth revolves through 360° 59′ 8″·33 in a mean solar day, express the length of a mean solar day (= 24 hours) in sidereal time (1871).

As 360° : 360° 59′ 8″·33 : : 24 : Ans. *Ans.* 24h. 3m. 56·5554sec.

49. What is meant by the hour angle of a heavenly body? Show how to find ship mean time from the hour angle of a star.

50. The hour angle of α Cygni (east of meridian) = 4h. 1m. 35sec., the star's right ascension = 20h. 36m. 54sec., and the right ascension of the mean sun 8h. 18m. 10sec.; find the ship mean time (1866). *Ans.* 8h. 17m. 9sec.

51. Construct a figure and show what is meant by the altitude of a heavenly body, its polar distance, zenith distance, hour angle, and azimuth or true bearing (1865).

52. Given the hour angle of a star: find the ship mean time (1865).

53. August 10, 1874, given the hour angle of α Tauri (Aldebaran) west of meridian = 3h. 35m. 27sec: find the ship mean time (1865).

R.A. of α Tauri, August 10d = 4h. 28m. 42·53sec.
R.A. of mean sun, ,, ,, = 9h. 15m. 11·02sec.

Ans. 22h. 48m. 58·51sec.

54. Define apparent solar time, mean solar time, and sidereal time, illustrating your definitions by projections (1) on the equinoctial; (2) on the horizon. In a sun chronometer what angle do you deduce from your observation combined with other data? and what must you apply to this before you can compare it with the time indicated by your chronometer (1869)?

55. What is meant by the hour angle of a heavenly body? How is the mean time found from the hour angle of the sun?

56. April 12, 1874, the hour angle of the sun W. of the meridian is found to be 75° 18′, what is the mean time (1867)?

Ans. April, 12d. 5h. 1m. 12sec.

57. At what time would Algenib pass the meridian 115° 28′ E. on August 20, 1874?

R.A. of Algenib = 0h. 6m. 47·14sec.
Sidereal time = 9h. 54m. 36·57sec.

Ans. 14h. 11m. 6·46sec.

58. Explain how to find the time when a heavenly body will pass the meridian.

59. March 20, 1874, in longitude 65° W., find what bright stars put down in the *Nautical Almanac* will pass the meridian between the hours of 9 and 12 P.M.

Sidereal time = 23h. 51m. 23″·41sec.
α Hydræ Constellation Leo, with Regulus.
ι Ursæ Majoris, or from ι to γ, Ursæ Majoris.

60. At what time will Markab cross the meridian of a place in latitude 47° 35′ N., longitude 120° W. on 20th September?

R.A. of Markab, 22h. 58m. 31·89sec.
Sidereal time, 11h. 56m. 49·75sec.

Ans. 10h. 58m. 34·58sec.

61. What bright stars in the *Nautical Almanac* passed the meridian of a place in longitude 54° 40′ E. between the hours of 7 and 10 on September 4, 1874 (1863)?

From γ Draconis to α Cygni the chief are Vega and Altair.
Sidereal time = 10h. 53m. 44·89sec.

62. Find the moon's distance from the earth when its horizontal parallax is 56′ 28″ (1865). *Ans.* 240882 miles.

63. What is the moon's distance from the earth when its horizontal parallax is 58′ 20″ (1865)? *Ans.* 233157 miles.

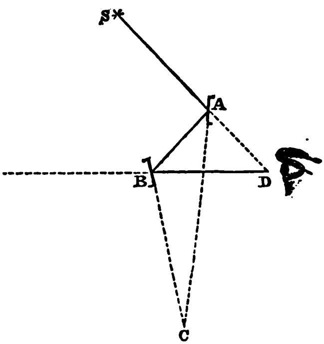
CHAPTER IV.

NAUTICAL INSTRUMENTS.

The Sextant—Its Optical Principle—Glasses—Horizon Glass—Index Glass—Telescope—The Limb—Index Bar—Vernier and Proof—Screws—Description—The Adjustments—Index Error—Rules to Find Index Error—Examples—Chronometer—Error of Chronometer—Compensation Balance—Dent's Compensation Balance—Rate of Chronometer—Reading of Chronometer—Civil and Astronomical Time—Examples—Examination Questions.

72. The Sextant and **Chronometer** are the two distinctive instruments used in Nautical Astronomy, in addition to the mariner's compass and azimuth compass described in the treatise on Navigation.

73. Sextant.—The sextant is an instrument employed for measuring the angular distances of objects by means of reflection. The chief angular distances the navigator has to find by it are the altitudes of the sun, moon, and stars, with the distances of the moon from certain celestial objects, in order to find the latitude or longitude. The optical principle of the construction of the sextant is, "that when a ray of light is twice reflected, the angle between the original ray of



light and the direction of its second reflection, is double the angle at which the reflecting surfaces are inclined."

Suppose A to be one reflecting surface, B the other, the angle of their inclination is B C A. If S A be a ray of light coming, say, from a star, it will be first reflected to B, then in the direction, suppose, of B D, meeting S A, produced in D, then angle

B D S is twice B C A.

This is the principle upon which the sextant, which we now proceed to explain, is constructed.

The chief parts of the sextant are (1) the glasses, (2) the limb, (3) the vernier, (4) the screws, (5) the microscope, (6) the telescope.

The Glasses are two, the *horizon glass* and *index glass*, with several shades or stained glasses interposed between these two to weaken the rays of light, and to deprive them of their calorific rays before they enter the eye. There are frequently four shades between the index and horizon glasses, and three in front of the horizon glass.

The *Index Glass* is silvered all over, and placed perpendicularly to the movable index limb. The image of the object is reflected from the index glass to the

Horizon Glass, which is a mirror perpendicular to the plane of the instrument, silvered only on the lower half, while the other half is unsilvered, and, therefore, transparent. After the ray of light has been reflected from the horizon glass, it passes into the

Telescope, which is a small ordinary telescope used to direct or strengthen, if we may use such a term, the visual ray from the observer's eye to the true image, as seen through the transparent part of the horizon glass, and the reflected image, as seen in the same glass.

In observing an object, the eye is placed at the eyepiece of the telescope, and directed towards the horizon; then the index limb is moved till the image of the object, star, or sun coincides with the horizon. The angle read from the limb is its altitude.

If two objects are observed, the telescope is directed so that one object can be seen through the unsilvered part; next the image of the second object is brought in contact with the other. The angle indicated on the limb is their angular distance.

The Limb is an arc of rather more than 60°. In fact the instrument is called a sextant because it is in the shape of a sector of a circle, the sector being rather more than one-sixth of a complete circle, made of iron, brass, etc. The limb is graduated from 0° to 140°, and so contrived as to enable the observer to read off at every 20″, 15″, or 10″. Although the sextant is, as its name implies, but the sixth part of a circle, because the ray of light is twice reflected we are enabled to read a little over twice sixty degrees upon the limb. The graduations of the limb are carried beyond zero to the right over a small space called the "arc of excess."

The *Radius* or *Index Bar* moves along the limb, and carries the index glass.

The Vernier is attached to the end of the index bar, and is used to subdivide the divisions on the limb into spaces of 10″, 15″, or 20″. It slides easily and evenly along the graduated limb, and enables the observer to read off his angles to within a very few seconds. The vernier is graduated in such a manner that the sixtieth of its divisions coincide with the fifty-ninth on the limb, or, generally, n divisions of the vernier are equal in length to $n-1$ on the limb.

Let L = the length of a division on the limb,
l = ,, ,, vernier,

$\therefore\ (n-1)\,L = n\,l$ (from above).

$\therefore\ l = \frac{n-1}{n}L$. Subtracting each side from L.

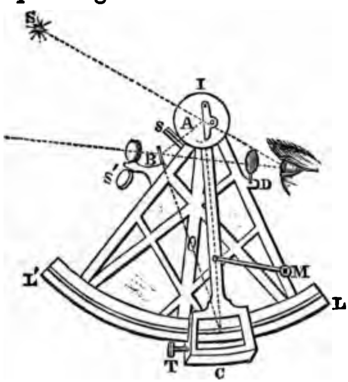
$$\therefore\ L - l = L - \frac{n-1}{n}L = \frac{nL - nL + L}{n} = \frac{L}{n}$$

or a division on the vernier is $\frac{L}{n}$ less than one on the limb. The best sextants have their arcs graduated

every 10 minutes, fifty-nine of these being equal to sixty on the vernier.

$$\therefore\ L - l = \frac{L}{n} = \frac{10'}{60'} = \tfrac{1}{6} \text{ of a minute of arc.}$$
$$= 10''.$$

Screws.—Two screws are used in the sextant, one to *clamp* the index bar when it has been brought approximately into its position, the other, the *tangent* screw, to give a very slight motion to the index bar after it has been clamped. There is also a milled-headed screw, which raises or lowers the rings by means of the up and down piece that carries the telescope, so that both the reflected and direct object can be seen through the telescope with equal brightness.



This figure is to represent the sextant. L L′ is the limb or arc, graduated from 0° to 120°, 130°, or 140°, from right to left or from L to L′. Towards L is the arc of excess, where it is graduated a little beyond the zero.

C is the vernier subdivided, as previously explained, to allow an arc to be read off to 10″ or 15″.

I C is the index bar, or radius bar, carrying the index glass I. The clamping screw is beneath C.

T is the tangent screw. While the clamping screw is to prevent the limb from moving, the tangent screw is to give it a very slight motion, to get the image and object in their proper places.

M is a microscope attached to the index bar by the arm seen in the figure. It is moved down over the vernier to enable the observer to read off correctly.

A or I is the index glass, which moves with the index bar, and *must* be fixed perpendicular to the plane of the instrument, and its face must be parallel to the length of the index bar.

B is the horizon glass, marked in the figure by two straight lines; the round objects in front are the shades. The index and horizon glasses must be parallel when the index stands at zero. As explained previously, the lower half only of the horizon glass is silvered.

s are the index shades; *s′* the horizon shades.

D is the up and down piece for raising or lowering the telescope, through the collar of which passes the telescope, not shown in the figure. The line B D, passing along the axis of the telescope into the eye, is called the line of collimation. It must pass parallel to the plane of the arc or sextant.

74. Adjustments of the Sextant are three:—

(1) *To adjust the index glass*, or to ascertain that it is perpendicular to the plane of the instrument. Place the index bar on about the centre of the limb, then, holding the sextant horizontally, bring it up to the eye, and looking over the limb obliquely into the index glass so as to see the limb by direct vision to the right and its reflection on the left. If the image appear in one and the same straight line with the arc itself, the index glass requires no adjustment; but if the image be not in a line with the arc, the sextant requires adjustment. Behind the index glass is a small screw. If the image be below the true arc, tighten this screw; if the image be above it, slacken it.

(2) *To adjust the horizon glass*, or to see that it is perpendicular to the plane of the instrument. This is done by looking steadily at the sun, and sweeping the index slowly along the limb, so as to make the reflected image pass over the direct image. If they do not exactly cover each other in passing, turn the screw, which is found immediately under the glass, until the one image exactly covers the other.

(3) The *third adjustment* is to adjust the line of *collimation.* The line forming the axis of the telescope is called the line of collimation: this must be *parallel* to the plane of the sextant. For this third adjustment the eye-piece of the telescope must be turned round until two of the wires in its focus appear parallel to the plane of the instrument. Now, bring the sun and moon, or moon and star, or two single stars—the objects chosen must be more than 90° apart—into apparent contact on the wire next the instrument; immediately, by altering the position of the sextant, raise them to the wire above; then, if they remain in contact, the telescope is parallel to the plane of the instrument; but if they separate, the screw farther away from the ring which holds the telescope to the up and down piece should be slackened, and the other tightened; but if they overlap, we must adjust in precisely the contrary manner.

75. Index Error.—However nicely constructed a sextant may be, by reason of expansion, contraction, etc., there is always a small amount of error in the parallelism of the index and horizon glasses. This is termed index *error.*

To find the index error.—When the sextant is properly adjusted in the three cases above referred to, then find the index error. Place the index bar at zero on the limb; a lozenge points out the exact place on the limb where the required zero will be found, or the lozenge must coincide with the zero. Then make the stroke upon the vernier indicating zero, by turning the tangent screw, to exactly coincide with the stroke upon the limb. When the index bar is placed at zero on the limb, see if the true and reflected images of the sun, moon, or other object appear to coincide exactly; if they do, there is no index error; but if they do not, move the index bar until the true and reflected images perfectly coincide—the *arc* through which the index bar has to be moved from zero is called the *index error.* Either of the following methods may be used to determine the amount of index error:—

(*a*) Make the true and reflected images of any distant and distinct object coincide—the best object for the purpose

is the horizon; then the index error will be the distance lying between 0° on the limb and 0° on the vernier.

If the reading be on the arc take it off, or call it minus –
,, off ,, put on ,, plus +

(*b*) Make the true and reflected images of the sun, by taking the sextant and looking directly at it, touch at the edges. Then the difference between the distances of 0° on the vernier and 0° on the limb, and twice the semi-diameter of the sun for the day, taken from the *Nautical Almanac*, is the index error.

When the reading is on the arc and greater, the error is –
,, ,, off ,, ,, ,, +
,, ,, on ,, less, ,, +
,, ,, off ,, ,, ,, –

1. May 12 day, 1874, the reading on the arc when the sun and its image was made to touch was 34′ 25″, find the index error.

May 12 day, 1874, semi-diameter of sun from *N. A.* = 15′ 51″·5
2

∴ Diameter of sun for May 12 day is .... 31′ 43″
Reading of the sextant *on* the arc .... 34′ 25″
∴ Index error .... –2′ 42″

It is subtractive, because the reading was on the arc, and greater than the *sun's* diameter.

2. May 13 day, 1874, the reading on the arc was 30′ 10″, when the true and reflected images of the sun were made to touch; required the index error.

May 13 day, 1874, semi-diameter of sun .... 15′ 51″·3
2

∴ Diameter of sun for May 13 day .... 31′ 42″·6
Reading of the sextant on the arc .... 30′ 10″
∴ Index error................. +1′ 32″·6

It is additive, because it is on the arc, and less than the sun's diameter.

3. On July 22 day, 1874, the images of the sun were just made to touch, and the reading *off* the arc was 33′ 45″; what was the index error?

July 22 day, 1874, semi-diameter of sun................15′ 46″·8
2

∴ Diameter of the sun............................ 31′ 33″·6
Reading of the sextant *off* the arc........... 33′ 45″
∴ Index error is .... + 2′ 11″·4

4. On July 23 day, 1874, the images of the sun were just made to touch, and the reading off the arc was 28′ 20″, required the index error.

July 23 day, 1874, semi-diameter of sun...............	15′ 46″·9
	2
∴ Diameter of the sun	31′ 33″·8
Reading of the sextant off the arc	28′ 20″
∴ Index error is	− 3′ 13″·8

(*c*) A third method of finding the *index error* is to bring the two images together, and make their edges touch first on one side then on the other. Illustrate this with two penny pieces: make the two edges touch, then slide one over the other till the opposite edges touch. When the two images of the sun touch in the two positions just indicated, take the readings in each case; then the index error is half their difference when one reading is on and the other off the arc:

additive when that off the arc is greater
subtractive ,, on ,, ,,

When both readings are on the same side of zero, half their sum is the index error:

additive when off the arc
subtractive ,, on ,,

5. October 1 day, 1874, the reading on the arc was 30′ 20″, when the sun and his image were made to touch, the semi-diameter being 16′ 1″·4, find the index error. *Ans.* + 1′ 42″·8.

6. October 2 day, 1874, the reading *off* the arc was 34′ 20″. when the sun and his image were made to touch, the semi-diameter being 16′ 1″·7, what is the index error?

Ans. + 2′ 16″·6.

7. On October 10 day, 1874, bringing the sun and its image together, the following readings were made to determine the index error of the sextant; find it and also the semi-diameter of the sun.

	For semi-diameter.		For index error.
Reading on the arc	33′ 10″		33′ 10″
Reading off the arc	31′ 10″		31′ 10″
	4)64′ 20″		2) 2′ 0″
Semi-diameter	16′ 5″	Index error	1′ 0″

The semi-diameter for the day in the *Nautical Almanac* is 16′ 3″·8, this shows that correct observations have been

made, and that we may rely upon the index error as being accurately determined.

8. Bringing the sun and its image together on December 10 day, 1874, the following readings were made to determine the index error; find its amount and the semi-diameter of the sun.

Readings on the arc 30′ 10″
Readings off the arc 35′ 0″

Ans. Semi-diameter 16′ 17″·5; index error + 2′ 25″.

9. To find the index error of a sextant I brought the two images together, and found the reading on the arc 35′ 20″, and off 29′ 40″; required index error and semi-diameter.

Ans. Semi-diameter 16′ 15″; index error − 2′ 50″.

10. On May 15d., 1874, the reading on the arc when the two images of the sun were in contact was 28′ 40″, what is the index error when the semi-diameter is 15′ 50″·9?

Ans. Index error + 3′ 1′·8.

11. May 1874, 1d., the reading off the arc when the sun and its image were in contact was 33′ 50′, given the semi-diameter 15′ 54″; what is the index error?

Ans. Index error + 2′ 2″.

12. May 1874, 31d., the reading off the arc when the sun and its image were in contact was 28′ 40″, find the index error when the semi-diameter is 15′ 48″·3. *Ans.* Index error − 2′ 56″·6.

13. Given the annexed observations; find the semi-diameter and index error August 15d., 1874.

On the arc	28′ 15″	Off the arc	35′ 0″
	29 45		33 45
	29 0		34 15
	87 0		103 0
	103 0		87 0
	12)190 0		6)16 0
Semi-diameter	15 50	Index error	+ 2 40

14. Given the following observations 1874, August 27d.; find the semi-diameter and index error:—

Reading on the arc 35′ 50″	Reading off the arc 27′ 20″
36 0	27 30
36 10	27 40

Ans. Semi-diameter 15′ 52″·5; index error − 4′15″.

15. Given the following readings off and on the arc; find the index error and semi-diameter for 1874, August 15d.

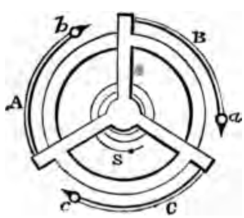
On the Arc.	Off the Arc.
30′ 40″	32′ 30″
31 0	32′ 30″

Ans. Semi-diameter 15′ 50″; index error + 50″.

76. Chronometer.—The chronometer is a very superior kind of watch or timekeeper. Sometimes they are made in the shape of a watch; but for sea purposes they differ materially in appearance from a watch, being much larger and kept in a cubical box of about six or eight inches cube. They are very sensitive, and are constructed so that with every change of temperature the balance is self-compensating. The value of a chronometer is determined by its capability of keeping uniform time, *whether it gain or lose is of no consequence so long as its rate of gain or loss is uniform.* The chronometer should be kept as near as possible to the line where there is the least motion in the ship, which is a line drawn from stem to stern through its centre of gravity. In the chronometer we have to draw attention to *error* and *rate.*

77. The Error of a Chronometer is the difference between the time shown by the chronometer and true Greenwich mean time; it is marked *fast* or *slow*, according as the chronometer is behind or before the Greenwich time. The chronometer is always supposed to show the time at Greenwich.

78. Compensation Balance for Chronometers.—If a bar be constructed of two metals of different expansibility, one firmly attached to the other, and the temperature rise, one of the two metals will expand more than the other; and the more expansible metal will form the outside or convex surface of the bar, and the least expansible the concave inner surface. On the contrary, should the temperature fall the opposite arrangement of the two metals will be made. The more expansible, which therefore *contracts* more, will form the concave surface, and the less expansible the convex outer surface, because if it expand less it will contract less.



In the construction of the balance wheel of the chronometer, these facts are practically applied. The balance is not made in one continuous rim (bar), but divided into three or four, generally three separate pieces, each fixed at one end, but free at the other. The more the pieces the more troublesome the compensation. The free ends are loaded with small weights. If each piece be composed of two metals—the more expansible being placed outside—then it is evident that if the outside, marked light in the figure, be the more expansible, upon an increase of temperature it will throw the small loaded ends towards the centre, while the whole balance itself will be thrown outwards; it is a great aim * of the balance maker so to apportion the parts of the machine, that as the substance of the wheel is moved by expansion from the centre, so the loads shall be thrown towards it. Likewise, should the piece of mechanism contract by cold, as the radius is shortened and the body thrown towards the centre, the loaded ends move outwards.

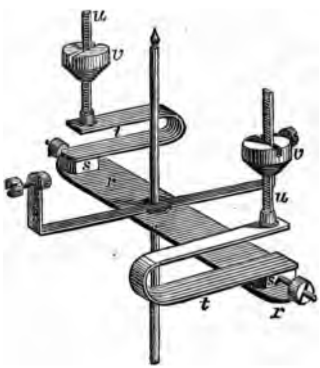
A B C is the compensated balance. A B C is the rim broken into three pieces, A, B, C. *a b c* are the loaded ends. *s* is a small hair spring. The dark lines forming A, B, C, indicate the less expansible of the two metals. The light lines forming A, B, C, indicate the more expansible of the two metals.

The metals generally employed are brass and steel. Such a mode of compensation is by no means perfect, since the time of the chronometer depends upon the moment of inertia of the balance, which varies not *directly as the distance* of the weight of the balance from its centre, but as the square of that distance. Hence a given amount of motion *from* the centre will produce a larger difference of time than the same amount to the centre, supposing

* The expansion of the balance is, as a cause for compensation, far inferior to the change of elasticity in the balance spring. Thus, in the error of an uncompensated watch, which amounts to 6½ minutes per day for 68° Fahrenheit, 1 minute 18 seconds is due to the expansion of the balance, and the remaining 5 minutes 12 seconds to the loss of elasticity in the spring.

the amount of expansion and contraction to take place from the same point.

79. Dent's Compensation Balance.—To overcome the difficulty referred to above, Mr. Dent, of 61 Strand, has constructed a compensation balance on a totally different principle. His principle of compensation is, that an ascent of temperature shall cause the compensation weights to move in *towards the centre* a *greater* distance, than the same descent of temperature causes them to move *outwards;* or, in other words, for a gradual rise of temperature the weights will move in faster and faster, and for a gradual fall slower and slower outwards.



This principle of compensation is effected thus:—*r* is a compensation diameter bar fixed on a balance axis composed of brass and steel, the steel being nearest the compensation weights. This is the primary compensation. *s s* are two blocks attached to the end of the bar to receive the secondary compensation pieces. *t t* two secondary compensation pieces, each made in the form of a staple, standing across the primary compensation bar at right angles to it, the brass being on the inside of the staples, the steel on the outsides. *u u* are two pillars fixed on the end of the

staples to carry the weights; they are furnished with screws, on which the weights turn for adjusting their heights. *v v* are two adjustable weights.

The compensation weights are made to move in the plane, passing through the axis of motion. On elevation of temperature, the distance between each arm of the staple is increased in height, and thus the compensation weight is raised from the balance bar; the augmentation thus effected by the secondary compensation enables the primary to carry the weight over a greater space and with accelerated velocity towards the centre of motion; the reverse effect, of course, taking place when the temperature falls.

To adjust the secondary compensation when the chronometer gains, the staples must be shortened or thickened, and the reverse if the chronometer lose. The primary compensation is adjusted by varying the height of the weights *v* on the screw *u*.

We will endeavour to make it a little clearer. The primary compensation is effected by the main bar *r r*, the secondary by the staples *t t*. When the main bar bends upwards by an increase of heat, it tilts the weights *v v* in towards the axis of motion; when it bends downwards by an increase of cold, it tilts them outwards. If the distance between *v* and *s* were constant, it would tilt the weights outwards very nearly the same distance as it tilts them inwards, and no advantage over the ordinary balance would be gained. The distance, however, between *v* and *s* varies by the bending of the staples with the temperature, for an increase of heat it is greater, for a decrease smaller; and it is obvious for any given tilt inwards, *the higher v is from r the greater will be the distance that v will be deflected inwards*, and for the same amount of tilt outwards, *the nearer v is to r the less will be the distance that v will be deflected outwards.* In other words, *v* will approach the centre for a given ascent of temperature more rapidly than it will recede from it for the same decrease.

80. Rate of a Chronometer is its gain or loss each day. It is generally but a second or two, and is determined by lunar observations, or by an astronomical clock, or by noticing the change in any given voyage, which is its *sea rate.*

81. Reading the Chronometer—Civil and Astronomical Time.—When at sea, and the master looks at his chronometer and finds it indicating a certain time, it is sometimes puzzling to know whether it be (say) 7 o'clock in the morning or the evening at Greenwich, in consequence of civilised nations in general having adopted the absurd custom of having the twenty-four hours of day and night divided into two twelves, instead of reckoning straight on from one to twenty-four o'clock. Common sense would tell us that if there are twenty-four hours in the day, we ought to have thirteen, fourteen, etc., up to twenty-four o'clock, the same as the Italians have.

Astronomical Time begins at 12 o'clock one day, and ends 24 hours after at noon next day. *Civil Time* begins at 12 o'clock at one night, and ends at midnight on the next. Astronomical time counts twenty-four hours; civil time two twelves.

Owing to this difference of civil and astronomical time, with the difficulty above alluded to, it is necessary to show clearly how at all times the exact date may be determined from the chronometer.

Take the civil date as May 31 day 8h. 30m. A.M.; the astronomical date corresponding to this is May 30 day 20h. 30m. It is at once seen that the chronometer cannot indicate such a date, but it ought to do so to avoid confusion. The following rule is deduced from what precedes :—

To find the astronomical date from the time shown by the chronometer.

If it be A.M. time, add 12 hours and call it the day before to obtain the astronomical date; but when the time is P.M. no alteration is necessary, since the chronometer then indicates astronomical time.

The difficulty, which arises from the ambiguity of the chronometer, may be overcome by taking the ship time and applying to it the longitude by account. This will give us the Greenwich date very nearly.

16. The ship time is 10 A.M. June 4 day, and the longitude by account is 20° 30′ E., what is the time at Greenwich?

Ship time,....June 3 day	22h. 0m.	Lon. in time.
Longitude E.,.....	1h. 22m.	20° 30′ E.
Green. date, June 3 day	20h. 38m.	4
		6,0)8,2 0
		1h. 22m. E.

∴ the 8 o'clock, as shown by the chronometer, is 20 hours, and not 8 hours.

17. The ship time is June 10 day 3h., the longitude by account is 125° W.; find the Greenwich hour indicated by the chronometer.

Ans. Chronometer shows 11h., etc.; this is P.M. ∴ the chronometer indicates the correct hour.

18. The ship time is June 12d. 6h., the longitude by account is 135° W.; what is the Greenwich hour indicated by the chronometer?

Ans. Chronometer shows 3h., etc.; this is 3 in the morning, or June 12 day 15h.

19. The ship time is June 12 day 6h., the longitude by account is 135° E.; what is the Greenwich hour indicated by the chronometer?

Ans. Chronometer shows 9h., etc.; this is 9 in the morning, or June 11 day 21h.

EXERCISES CHIEFLY FROM EXAMINATION PAPERS.

20. State the errors to which the sextant is most liable, and the methods of detecting and correcting them (1861).

21. Give the mathematical principles of the vernier (1861).

22. Give the geometrical principles of the sextant (1861).

23. Why on a sextant, with an arc of 60°, do we find 120° marked upon it (1862)?

24. What is the index error of a sextant? How is it obtained with the greatest accuracy? and what determines its sign, plus or minus (1863, 1865)?

25. Explain the construction and use of the sextant (1864). What are the adjustments of a sextant? Show how to find the index error by measuring the diameter of the sun on and off the arc (1871).

26. The reading on the arc is 31′ 20″, and that off the arc is 33′ 40″; what is amount and algebraical sign of the index error (1871)? *Ans.* Index error + 1′ 10″.

27. In the sextant the index correction is half the difference of the sun's diameter measured off and on the arc; required a proof (1866).

28. State the principle of the sextant. How large an angle can I measure with a sextant whose limb contains 70°? What do you mean by the index error? Show how to find it (1869). *Ans.* 140°.

29. What is a chronometer? Describe one.

30. To what errors is the chronometer liable? and show clearly a method of compensating for these errors.

31. What do you mean by the rate of a chronometer? How are chronometers rated?

32. To what ambiguity does our present mode of counting time subject us? Show by illustrations the difference between civil and astronomical time.

CHAPTER V.

ALTITUDES—CORRECTIONS.

Definitions—Dip—Proof for Dip—Table of Dip—Examples—Refraction—Proof for Refraction—Illustration—Parallax—Parallax in Altitude—Horizontal Parallax—Proof for Parallax.

82. **The Altitude** of a celestial object is its height above the horizon, or it is the arc of a circle of altitude passing over the body which is intercepted between the object and the horizon.

83. **The Observed Altitude** is the altitude as measured by the sextant.

84. **Apparent Altitude** is the altitude as it appears to the observer, or it is the observed altitude corrected for index error and dip.

85. **True Altitude** is the observed altitude corrected for refraction, parallax, index error, and dip, or the altitude as seen from the centre of the earth.

86. **The True Place** of a heavenly body is the position in which it would appear if seen from the centre of the earth.

87. **The Apparent Place** of a heavenly body is its position as seen from the position of the spectator.

The altitudes of the sun, moon, etc., as determined by the sextant, are liable to error on account of the (1) *index error* of the sextant; (2) because the *eye* of the observer is *above the surface of the earth*; (3) on account of the *effects of the atmosphere* upon the rays of light passing through it from the object to the observer's eye; (4) also it is very difficult to properly *obtain the centre of the sun* in an observation; and (5) the observer is at the *surface* of the earth, and not at the *centre*.

(1) The *Index error* has already been discussed.

(2) **The Dip.**—The higher we rise above the earth the more of its surface is exposed to our view, the more does its vast size dawn upon us, and, if near the sea, the more apparent becomes its rotundity. If, then, as we ascend more objects come into view, it is quite evident that when we see a celestial object from any altitude above the sea, our observed altitude will be too great. The height of an object above the horizon of any given place on the earth's surface, is greater than its true height above the horizon by the angle subtended by a horizontal line from the spectator's eye, and a tangent to the earth's surface from the place of observation.

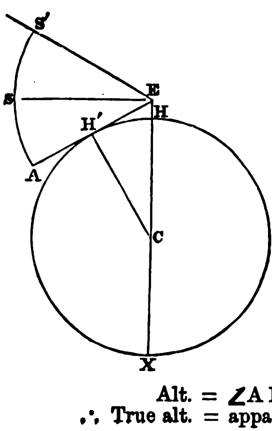
Dip is the correction to be made in any observed altitude, on account of the sphericity of the earth and the height of the spectator.

Proof for Dip—

Dip in minutes $= \cdot 9752\sqrt{h}$, h being reckoned in feet.

In the annexed figure, suppose H′HX is the meridian of the observer.

Let C be the centre of the earth, HE the observer.



Draw SE perpendicular to EH from the observer's eye. Let S′ be the sun, or other celestial object; then its apparent altitude to the observer is the arc AS′ or angle AES′. The dip is the angle SEA; the true altitude SES′.

It is very evident from the above, and by an inspection of the figure, that the following equation always holds good:

$$\text{Alt.} = \angle \text{AES}' - \text{SEA}.$$

$$\therefore \text{True alt.} = \text{apparent alt.} - \text{dip}.$$

Join H′ C, when angle C H′ E is a right angle,

$\therefore$ H′ C E and H′ E C = a right angle (*a*)

But S E C is a right angle,

$\therefore$ S E H′ + H′ E C = a right angle.

$\therefore$ S E H′ + H′ E C = H′ C E + H′ E C (from *a*).

Omit the common angle H′ E C, then

$$\text{S E H}' = \text{H}'\text{C E}.$$

$\therefore$ the $\angle$H′ C E at the centre of the earth is the dip.

$$\text{Cos. H}'\text{C E} = \frac{\text{H}'\text{C}}{\text{C E}} = \frac{\text{H}'\text{C}}{\text{H C}+\text{H E}}, \text{ but H}'\text{C} = \text{H C} = r$$

$$\therefore \text{Cos. H}'\text{C E} = \frac{r}{r+\text{H E}}$$

Subtract each side from unity, we have

$$1 - \text{Cos. H}'\text{C E} = \frac{r+\text{H E}-r}{r+\text{H E}} \text{ (let H E} = h)$$

$$1 - \text{Cos. H}'\text{C E} = \frac{h}{r+h}$$

Let $\angle$H′C E be called C; and since *h* is very very small compared with the radius of the earth, we may neglect it in the large denominator, and write the equation thus—

$$1 - \text{Cos. C} = \frac{h}{r}$$

$$\text{but} \because \text{Cos. C} = 1 - 2\,\text{Sin.}^2 \frac{\text{C}}{2}$$

$$\therefore 2\,\text{Sin.}^2 \frac{\text{C}}{2} = \frac{h}{r}$$

$$\therefore 4\,\text{Sin.}^2 \frac{\text{C}}{2} = \frac{2h}{r}$$

Extracting the square root,

$$2\,\text{Sin.} \frac{\text{C}}{2} = \sqrt{h} \times \sqrt{\frac{2}{r}}$$

$$\text{or, Sin.} \frac{\text{C}}{2} = \sqrt{h} \times \sqrt{\frac{1}{2r}}$$

The angle $\frac{\text{C}}{2}$ being very small, we may write $\frac{\text{C}}{2} \times$ Sin. 1′ for it (C being minutes of arc),

$$\therefore \frac{\text{C}}{2} \times \text{Sin. } 1' = \sqrt{h} \times \sqrt{\frac{1}{2r}}$$

$$\therefore \text{C} = \sqrt{h} \times \sqrt{\frac{2}{r}} \times \frac{1}{\text{Sin. } 1'} = \text{dip in minutes of arc.}$$

By substituting the proper value for r and Sin. 1′,

$$\text{Dip in minutes} = \sqrt{h} \times 1{\cdot}0639$$

now making due allowance, about $\frac{1}{13}$ for refraction,

$$\text{Dip in minutes} = \sqrt{h} \times \tfrac{12}{13} \times 1{\cdot}0639$$
$$= {\cdot}9752 \sqrt{h}$$

thus we see that the dip in minutes equals the square root of the height very nearly.

TABLE OF DIP USED THROUGHOUT THIS BOOK.

Height of Eye.	Dip− ′	″	Height of Eye.	Dip− ′	″	Height of Eye.	Dip− ′	″
1	0	58·5	26	4	58	51	6	57
2	1	22·7	27	5	2·8	52	7	2
3	1	41	28	5	9·6	53	7	6
4	1	57	29	5	15	54	7	10
5	2	10·8	30	5	20·5	55	7	13·9
6	2	23	31	5	25·7	56	7	17·8
7	2	34·8	32	5	31	57	7	21·7
8	2	45·4	33	5	36	58	7	25·6
9	2	55·5	34	5	41	59	7	29
10	3	4·9	35	5	46·7	60	7	33
11	3	14	36	5	51	65	7	51·7
12	3	22	37	5	55·8	70	8	8·8
13	3	31	38	6	0·6	75	8	26·7
14	3	38·8	39	6	4·9	80	8	45·5
15	3	46·6	40	6	10	85	8	59
16	3	54	41	6	14·6	90	9	15
17	4	1	42	6	19	95	9	30
18	4	8	43	6	21·6	100	9	45
19	4	15	44	6	28	110	10	13·8
20	4	21·7	45	6	32	120	10	40·8
21	4	28	46	6	38	130	11	7
22	4	34	47	6	41·7	140	11	31·8
23	4	40	48	6	45	150	11	56
24	4	47	49	6	49·5	160	12	19·8
25	4	52·5	50	6	53·7	200	13	47

Dip has always to be subtracted, because when the eye is above the level of the sea, we measure the altitude of

an object from a little below the sensible horizon; or the altitude that we measure with a sextant is greater than it should be, because the observer brings the image down to his horizon, which is lower than the horizon made by drawing a tangent to the surface of the sea immediately beneath the feet. In the figure the height H E of the observer is a vast deal greater than it should be in comparison with HX, the diameter of the earth, and, therefore, the angle S E A′ is distorted to a size immensely greater than the real angle.

Find the dip for an altitude of 200 feet.

$$\text{Dip} = \cdot 9752\sqrt{h}$$

$$\begin{array}{rcl} \text{Log. } 200 & = & 2)\underline{2\cdot301030} \\ & & 1\cdot150515 \\ \text{Log. } \cdot9752 & = & \underline{\bar{1}\cdot989094} \\ \text{Log. } 13\cdot79 & = & 1\cdot139609 \\ \underline{60} & & \\ 47\cdot40 & & \end{array}$$

Ans. Dip 13′ 47″.

Logarithm of 200 was taken from the tables and divided by 2 to find the square root, then log. ·9752 was added to it. This gives log. 13′·79, or the dip in minutes.

Let the student for practice take any of the heights given in the table and find the dip. It will be found a very good exercise by which to obtain a knowledge of the tables. We give one more illustration.

Given the height of the eye above the level of the sea 85 feet; what is the amount of dip?

$$\begin{array}{rcl} \text{Log. } 85 & = & 2)\underline{1.929419} \\ & & \cdot964709 \\ \text{Log. } \cdot9752 & = & \underline{\bar{1}\cdot989094} \\ \text{Log. } 8\cdot991 & = & \cdot953803 \\ \underline{60} & & \\ 59\cdot460 & & \end{array}$$

Ans. Dip 8′ 59″.

(3.) **Refraction.**—Refraction is the allowance made in an observed altitude consequent upon the deflection of the rays of light which come from the celestial object to the observer's eye. Its amount depends upon the

density and quantity of the different strata of air through which the rays of light pass. Hence the refraction is greatest when an object is in the horizon, while it gradually diminishes to the zenith, where it is *nil.*

The density of the air gradually increases from its highest elevations to its greatest depths. We may suppose it to consist of layers lying one upon the other, each of which insensibly increases the amount of refraction, so that when we consider the strata as indefinite in number, we shall get the ray of light in a curve; and as we always see an object in the direction in which the ray of light last comes to the eye, the object will be seen in a tangential direction to the curve where it enters the eye. Hence the object is seen higher than it really is, so that refraction *has always to be subtracted.*

We may now more clearly state how refraction is measured. Refraction is measured by the angle between the tangent to the curve thus formed from the observer's eye, and the direction of a ray as it would have come to the observer uninfluenced by the varying density of the atmosphere.

Proof that Refraction

$=57''$ Tan. zen. dis. (nearly.)

The sine of the angle of incidence with which a ray of light falls upon a body, always bears a constant ratio to the sine of the angle of refraction when the ray passes from one medium to another. This is called the index of refraction, and varies with the medium, but is always the same for the same medium.

Let m be the mean refraction.
r ,, constant ratio.
z ,, angle of refraction.
$\therefore\ z+m$ is the angle of incidence.

Hence we shall have from the index of refraction—

$$\text{Sin. }(z+m)=r\text{ Sin. }z.$$
$$\therefore\ \text{Sin. }z\text{ Cos. }m+\text{Cos. }z\text{ Sin. }m=r\text{ Sin. }z.$$

The mean refraction when at a maximum is only 33 or

35 minutes, and, therefore, without introducing any perceptible error, we may take

$$\text{Cos. } m = \text{Cos. } 0 = 1, \text{ and Sin. } m = m'' \text{ Sin } 1''.$$

$$\text{Hence} \quad \text{Sin. } z \text{ Cos. } m + \text{Cos. } z \text{ Sin. } m = r \text{ Sin. } z$$

$$\text{becomes} \quad \text{Sin. } z + m'' \text{ Sin } 1'' \text{ Cos. } z = r \text{ Sin. } z.$$

$$\therefore m'' \text{ Sin. } 1'' \text{ Cos. } z = r \text{ Sin. } z - \text{Sin. } z.$$

$$= \text{Sin. } z\,(r-1).$$

$$\therefore m'' = \frac{\text{Sin. } z}{\text{Cos. } z} \cdot \frac{r-1}{\text{Sin. } 1''}$$

$$= \text{Tan. } z \quad \frac{r-1}{\text{Sin. } 1''}$$

It is found that after allowing for the temperature, density, etc., of the atmosphere, $\frac{r-1}{\text{Sin. } 1''}$ may be taken as 57″ for a constant quantity, and as z is the zenith distance, or may be taken for the zenith distance, therefore

Refraction in seconds = 57″ Tan. *zen. dis.* (nearly).

In consequence of the refraction changing very irregularly, when an object is near the horizon this formula does not hold good for altitudes under 12° or 13°.

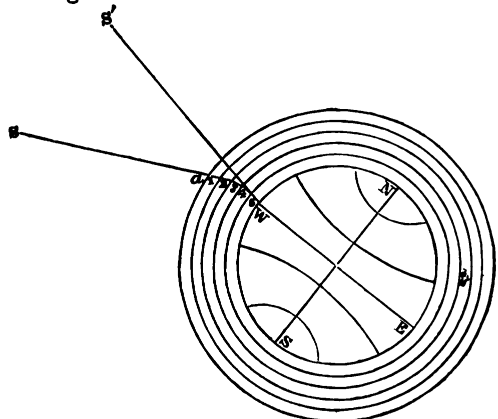


Illustration of Refraction.—Let N W S E represent the earth; W the position of a spectator on its surface;

S an object seen by a person stationed at W; the circles around the earth different strata of air.

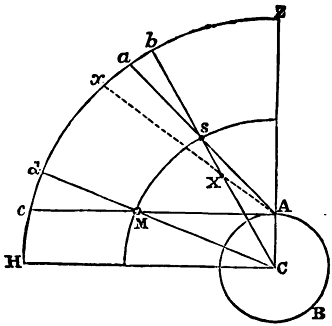
A ray of light starting from the object S enters the atmosphere at *a*, and passes through the first layer, being refracted; it is again refracted in the second, third, etc., till it reaches the spectator's eye at W; the spectator sees the object in the direction in which the ray last comes to his eye. Hence he sees it at S′ in the direction WS′.

The object appears to be higher than it really is, so the amount of refraction has always to be subtracted.

(4) **Parallax.**—Parallax of the sun is the apparent size of half the diameter of the earth, as it would be seen from the sun's centre, or half the angle between two lines drawn from the sun to the outside of the earth at the equator, which manifestly varies inversely as the sun's distance. According to the latest average of observations, the sun's parallax is 8″·943, and not 8″·57 as formerly given. It is by using 8″. 943 for the parallax that has given 91,404,000, which is the mean of 89,860,000 and 92,950,000, as the sun's distance, instead of 95,383,000 miles. The parallax of the sun has been determined by the transit of the planet Venus.

The moon's parallax is the apparent semi-diameter of the earth as seen from the moon; the moon's mean parallax is 57′ 2″. The above definitions of parallax, and the following illustrations, are taken from Denison's *Astronomy without Mathematics*. Two observers, one on one side of the earth, the other on the opposite, as far apart as they can be, will, if they both look at the moon, see her apparently covering two different places among the stars behind her, as a post covers different places in the wall behind it when seen by different observers standing apart; or she will appear at a different distance from the same star that may be quite close to her; this distance is measured by the angle between the two lines from the observer to the moon's centre and the star. The difference

between the angles as seen by the two observers is the parallax.



Suppose AB to be the earth, C its centre, A the position of a spectator, Z his zenith, *s* the position of the moon, Mercury, etc. A person standing at A would see the object *s* on the celestial concave at *a*, while to a person at the centre of the earth, it would seem to be at *b*; the arc *ab*, or the angle *asb*, or A*s*C, is the parallax.

When an object is supposed to be seen from C, the centre of the earth, its position is called its true or geocentric place. Hence *b* is the true or geocentric place of *s*. When the radius of the earth is taken as the base of the triangle *s*AC, then *asb* is the *diurnal* parallax or *parallax in altitude.*

When the body, as M, is in the horizon, then angle *d*M*c*, or AMC, or arc *dc*, is the *horizontal parallax.*

It is clearly manifest by closely inspecting the figure, that the parallax of an object is greatest when it is in the horizon, and that it gradually diminishes as the body rises in the sky, until it arrives at the zenith, where the parallax vanishes, the two lines C*b* and A*a* coinciding with CZ.

It is also obvious that the altitude of an object as seen from the earth's surface, is less than it would be if seen from the earth's centre, hence parallax is always added to the apparent altitude to find the true altitude.

Parallax depends upon the distance of an object from the spectator; the nearer the object the greater its parallax with a given movement. Let X be the place of an object much nearer than *s*, then its parallax *bx*, or angle

bXx, is much greater than ab or angle bsa. The moon being near the earth, its parallax is greater than that of any other heavenly body. The stars are at such an immense distance, that they may almost be said to have no parallax.

A familiar illustration is afforded by the following fact; If a fire or a light be a certain distance away, and we move at right angles to it, and it changes its position, it is near; but, if we move a long way, and it retains its position, it is a great distance away.

Proof that

$$para.\ in\ alt. = horizontal\ parallax \times \text{Sin. } app.\ zen.\ dis.$$

In the last figure

A s C is the parallax in altitude $= p$
A M C ,, horizontal parallax $= h$
C s or C M is the distance of the body $= d$
Z A a is the apparent zenith distance.

In triangle A s C we have

$$\frac{AC}{Cs} = \frac{r}{d} = \frac{\text{Sin. } AsC}{\text{Sin. } ZAa^*} = \frac{\text{Sin. } p}{\textit{Sin. app. zen. dis.}} \quad (1)$$

Again, from the right-angled triangle C A M—

$$\text{Sin. CMA or Sin. } hor.\ para. = \frac{AC}{CM} = \frac{r}{d} \quad (2)$$

Therefore, from (1) and (2) we have the equation

$$\text{Sin. } hor.\ para. = \frac{\text{Sin. } p}{\text{Sin. } app.\ zen.\ dis.} \quad \because \text{ each} = \frac{r}{d}$$

$$\therefore \text{Sin. } p = \text{Sin. } hor.\ para. \times \text{Sin. } app.\ zen.\ dis.$$

But since parallax, even in the case of the moon, is always very small

$$\therefore \text{Sin. } p = p'' \text{ Sin. } 1'', \text{ and Sin. } hor.\ para. = hor.\ para.'' \times \text{Sin. } 1''$$
$$\therefore p'' \text{ Sin. } 1'' = hor.\ para.'' \text{ Sin. } 1'' \times \text{Sin. } app.\ zen.\ dis. \text{ (cancelling).}$$
$$\therefore p \text{ or } para.\ in\ alt. = hor.\ para. \times \text{Sin.} app.\ zen.\ dis.$$

The positions on the celestial sphere of the sun, stars, etc., on the one hand, and the moon alone on the other, are a little different to what we should expect looking at

* sAC ∵ Sin. of an angle = Sin. of its supplement.

them superficially. The moon, in consequence of the parallax exceeding the refraction, always appears *lower* in the sky than its true geocentric place, but the sun and other heavenly bodies always appear higher than their true geocentric position.

EXERCISES CHIEFLY FROM EXAMINATION PAPERS.

1. What is meant by correction for dip? Give a formula for calculating its amount (1866).

2. What is meant by the correction for refraction? How does it affect the observed altitude of a heavenly body? Obtain an expression for calculating the refraction for any zenith distance, explaining the meaning of each of the terms? Within what limits is the expression to be relied on (1866)?

3. What is meant by the correction for refraction? How is it applied to the observed altitude to obtain the true altitude?

4. The observed altitude of Regulus was 36° 15′ 12″; what is the altitude corrected for refraction (1866, 1867)?

Ans. 36° 13′ 54″·5.

5. What is a parallax? How does it affect the apparent position of a heavenly body? What is meant by the augmentation of the moon's semi-diameter (1867)?

6. Show how to correct an observed altitude of a heavenly body for dip (1863).

7. Prove that horizontal parallax × Cos. apparent alt. = parallax in alt. (1863).

8. Investigate an expression for computing the parallax in altitude of a heavenly body (1864).

9. Explain what is meant by correction for dip. Calculate a formula for it (1865).

10. What is meant by the correction of parallax? Given the apparent reduced zenith distance and horizontal parallax: show how to calculate the diurnal parallax, and thence the true reduced zenith distance (1865).

CHAPTER VI.

DECLINATION AND EQUATION OF TIME.

Definition—Explanation—Correction—Hourly Difference—Examples—Equation of Time Vanishes Four Times a Year—Correction—Examples.

88. **The Declination** of an object is its angular distance north or south of the equator.

The declination of the sun changes every moment. It is always either increasing or decreasing. The sun crosses the equator on the 20th day of March, 1874, when its declination is *nil;* from that instant its north declination gradually increases till the 21st day of June, from which date it decreases till the 22nd day of September, when the sun crosses the equinoctial line, and we have south declination, which increases till the 21st day of December, after which it decreases till the 20th day of March.

On the 21st day of June sun's declination is 23° 27′ 27″·5 N.
On „ 21st „ Sept. „ „ „ 23° 27′ 24″·2 S.

On pages I. and II. for each month in the *Nautical Almanac* will be found the declination of the sun for each day of the month *at noon;* on page I. for apparent noon, on page II. for mean noon; also on page I., in the sixth column, is found the difference for one hour. This difference for one hour has to be applied to the declination both for apparent and mean noon, to find the correct declination for any time preceding or following noon.

Suppose the "difference for one hour" is 20″·32, and we want the declination for three in the afternoon, when the declination is *increasing,* we have to multiply 20″·32

by $3 = 60''\cdot92$, which is the difference for three hours, this being added to the declination for noon gives the declination for three hours after noon; it is evident that had the declination for three hours *before* noon been wanted, we must subtract this "difference" to obtain the true declination.

1. The declination for mean noon June 10 day, 1874, is 23° 1′ 54″·4 N., increasing the hourly difference 11″·44; find the declination for June 9 day 18h. 24m., and for June 10 day 5h. 36m.

It is seen that 18h. 24m. is the same time *before* noon that 5h. 36m. is *after* noon.

Dec. for June 9 day 18h. 24m.	H. D.
Dec. 23° 1′ 54″·4 N.	11″·44
−1′ 4″·06	5·6
Dec. 23° 0′ 50″·34 N.	6864
	5720
	64·064

Dec. for June 10 day 5h. 36m.
23° 1′ 54″·4 N.
+1′ 4″·06
Dec. 23° 2′ 58″·46 N.

The hourly difference (H. D.) is put down, this is multiplied by 5·6, because 5h. 36m. = 5·6 hours: the result is 64″·06 to be applied to the declination.

The 64″·06 is subtracted to find the declination for June 9 day 18h. 24m., because as the declination is increasing 5·6 hours *before* noon, *it must have been less* than at noon, and certainly 5·6 hours *after* noon it will *be more* if *increasing*, hence to find it for June 10 day 5h. 36m. the H. D. is added.

To correct the sun's declination.—(1) Take out the declination for the *nearest* noon to the Greenwich date, and the hourly difference.

If you are using mean time, take out the declination for *mean noon*, if apparent time for *apparent noon.*

(2) Multiply the hourly difference by the number of hours before or after noon, taking care to reduce the hours and minutes to the decimal of an hour.

Or else multiply by the hours, and take parts for the minutes and seconds.

(3) If the declination is increasing, add if you go forward, that is, if it be P.M. time; subtract if it be A.M. time, or if you go back.

If the declination is decreasing, subtract if it be P.M. time, add if A.M. time.

2. The declination for apparent noon 25th November, 1874, is 20° 47′ 28″·1 S., increasing the hourly difference 29″·26; find the declination for Nov. 25 day 4h. 10m., and for Nov. 24 day 19h. 50m. apparent time.

As before 19h. 50m. is the same time before noon that 4h. 10m. is after noon.

Dec. for 24 day 19h. 50m.
20° 47′ 48″·1 S.
2′ 1″·91
Dec. 20° 45′ 46″·19 S.

H. D.
10 = ⅙ 29″·26
4
117″·04
4″·87
6,0)12,1″·91
2′ 1″·91

Dec. for 25 day 4h. 10m.
20° 47′ 28″·1 S.
0° 2′ 1″·91
Dec. 20° 49′ 30″·01 S.

3. What is the corrected declination for mean time, January 31 day 18h. 42m., and for February 1 day 6h. 12m., when the declination for February 1 day mean noon is 17° 3′ 38″ S. decreasing, and hourly difference 42′ 81″?

Jan. 31 day 18h. 42m.
Feb. 1 day, dec. 17° 3′ 38″ S.
0° 3′ 49″·89
17° 7′ 27″·89 S.

H. D.
42″·81 — 42″·81
5·3 — 6·2
12843 — 8562
21405 — 25686
6,0)22,6·893 — 6,0)26,5·422
3′ 46″·89 — 4′ 25″·4

February 1 day 6h. 12m.
17° 3′ 38″ S.
0° 4′ 25″·4
16° 59′ 12″·6

4. Find the corrected declination for mean time 1874, June 12 day 1h. 20m. 10sec., when the declination for mean noon is 23° 10′ 14″·8 N. increasing, hourly difference 9·41sec.

Ans. 23° 10′ 27″ N.

5. Find the corrected declination for mean time 1874 Nov. 9 day 17h. 30m. 46sec., when the declination for mean noon Nov. 10 day is 17° 11′ 45″·2 S. increasing, hourly difference 42·14 seconds. *Ans.* 17° 7′ 12″·6 S.

6. Find the corrected declination for apparent time at Greenwich 1874, February 13 day 19h. 54m. 17·6sec., when the declination for apparent noon February 14 day is 12° 58′ 2″·6 decreasing, hourly difference 51·11 seconds. *Ans.* 13° 1′ 31″·87 S.

7. Required the corrected declination for apparent time at Greenwich 1874, July 10 day 2h. 22m. 6sec., when the declination for apparent noon is 22° 14′ 50″·5 N. decreasing, hourly difference 19 seconds. *Ans.* 22° 14′ 6″·3 N.

8. What is the corrected declination for mean time at Greenwich 1874, May 13 day 8h. 14m. 25sec., when the declination for mean noon is 18° 24′ 45″·3 N. increasing, hourly difference 36·95 seconds? *Ans.* 18° 30′ 15″·4 N.

9. What is the corrected declination for mean time at Greenwich 1874, May 12 day 19h. 20m., when the declination for mean noon, May 13, is 18° 24′ 45″·3 increasing, hourly difference 36·95 seconds? *Ans.* 18° 21′ 52″·88 N.

10. Required the corrected declination for 1874, January 21 day 9h. 20m. 14sec. mean time at Greenwich, when the declination for mean noon is 19° 52′ 27″·9 S. decreasing, hourly difference 33·62 seconds? *Ans.* 19° 47′ 14″ S.

11. What is the corrected declination for 1874, January 20 day, 19h. 19m. 53sec mean time at Greenwich, when the declination for mean noon January 21 day is 19° 52′ 27″·9 S. decreasing, hourly difference 33·62 seconds? *Ans.* 19° 55′ 4″·8 S.

89. **Equation of Time** is the difference between apparent time and mean solar time. This equation of time vanishes four times in the year: in the year 1874 on April 15 day, June 14 day, August 31 day, December 24 day. On November 3 day, the true solar time is 16′ 19″·21 before mean or clock time; but on February 11 day it is 14′ 29″·18 behind the clock time. It may have been observed that the afternoons appear longer directly after Christmas than before. We have here the reason for it in the difference between sun time and clock or mean time. The causes of this inequality of the solar days are—

(1) The irregularity of the sun's motion in his path, resulting from the elliptic form of the earth's orbit.

(2) The inequality of the angles through which the

meridian must revolve on successive days to overtake the sun, caused by the obliquity of its path.

These two causes are expressed more simply thus—

(1) The unequal velocity of the sun in his apparent orbit.

(2) The unequal motion of the sun in consequence of the obliquity of the ecliptic to the equator, or in consequence of the sun moving in the ecliptic and not in the equator.

Practically, the sun moves through the same path every four years; hence the equation of time is nearly the same every four years, so are also the times of sunrise and sunset. The equation of time is found on the I. and II. page of each month in the *Nautical Almanac*, with directions as to its application. On page I. is stated whether it is to be added to or subtracted from apparent time, and on page II. is stated whether the equation of time is additive to or subtractive from mean time. The correction is found on page I., right hand column, and is applied and found in exactly the same way as that for declination. In fact, the rules for the correction of equation of time are precisely the same as those for declination.

12. The equation of time for January 12 day 1874 is 8m. 37·34sec., increasing subtractive from mean time; find that for January 12 day 3h. 24m., when the hourly difference is ·959sec.; also find the apparent time.

Equa. of Time.	Correction.	Apparent Time.	
m. sec.	sec.		h. m. sec.
8 37·34	·959	M. T. Jan. 12 day...	3 24 0
3·26	$3\frac{2}{5}$	Equation of time.....	−8 40·6
8 40·6	2·877	A. T. Jan. 12 day...	3 15 19·4
	·383		
	3·260		

13. The equation of time for 1874 Nov. 16 day is 15m. 3·38sec. decreasing and to be added to mean time; find that for Nov. 15 day 20h. 17m. 12sec., when the hourly difference is ·469 sec., also find the apparent time.

Equa. of Time.	Correction.		Apparent Time.			
m. sec.		sec.		h.	m.	sec.
15 3·38	$30=\frac{1}{2}$	·469	M. T. Nov. 15 day...	20	17	12
1·74	$12=\frac{1}{5}$	3			+15	5·2
15 5·12		1·407	A. T. Nov. 15 day...	20	32	17·2
		·234				
	$48=\frac{1}{15}$	·093				
		·006				
		1·740				

14. Find the corrected equation of time for 1874 Nov. 16 day 1h. 51m. 24sec. ; also the apparent time at place when the equation of time for Nov. 16 day at noon is 15m. 3·38sec. additive decreasing, hourly difference ·469sec.

Ans. { Eq. of time, 15m. 2·51 sec.
A. T., 2h. 6m. 26·51 sec.

15. Find the corrected equation of time for Greenwich mean time 1874 January 12 day 1h. 56m. 40sec., that for mean noon being 0m. 31·24sec. decreasing, hourly difference ·515sec. subtraction from mean time. *Ans.* 0m. 30·3sec.

16. Find the corrected equation of time for Nov. 9 day 18h. 25m. 14sec. when the equation of time for mean noon Nov. 10 day is 15m. 56·08sec. decreasing and subtractive, and the hourly difference ·261sec. *Ans.* −15m. 57·53sec.

17. What is meant by equation of time? To what causes is it due?

1874 January 15 day, in longitude 27° 20′ W., the apparent time at ship is 8h. 25m. 13sec. P.M., what is the mean time (1867)?

Greenwich Date.				Longitude in Time.
A. T. at ship, Jan. 15 day...	8	25	13	27° 20′ W.
	1	49	20	4
Jan. 15 day...	10	14	33	6,0)10,9 20
				1h. 49m. 20 sec.

Equa. of Time.	Correction.		Mean Time at Ship.			
m. sec.		sec.		h.	m.	sec.
9 43·74	$10=\frac{1}{6}$	+ ·88	A. T. Jan. 15 day...	8	25	13
9·0		10			+9	52·7
9 52·7		8·80	M. T. Jan. 15 day...	8	35	5·7
	$4\frac{1}{15}$	·146				
	$30=\frac{1}{8}$	·058				
	$3\frac{1}{10}$	·007				
		·000				
		9·011				

18. 1874 June 10 day, in longitude 47° 18′ 20″ E., the apparent time at place is 11h. 10m. A.M., what is the mean time and equation of time when equation of time for apparent noon June 10 day is 0m. 55·52sec. subtractive, decreasing, difference for one hour ·496sec. ?

Ans. Eq. of time, 0m. 57·5 sec.
M. T., 11h. 9m. 2·5 sec.

19. 1874 July 21 day, in longitude 150° 18′ 40″ W., the mean time at place is 10h. 28m. 15sec. P.M., find the equation of time and apparent time at place.

$$\text{Equation}\begin{cases}21\text{ day} = 6\text{m. } 5{\cdot}75\text{sec.}\\ 22\text{ day} = 6\text{m. } 8{\cdot}45\text{sec.}\end{cases}\text{Subtractive}\begin{cases}\text{diff. } {\cdot}125\\ \quad,, \quad {\cdot}100\end{cases}$$

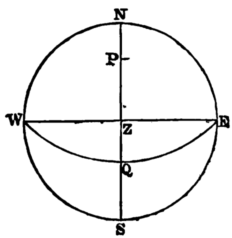
Ans. Eq. of time, 6m. 8·1sec.
Ap. time, 10h. 22m. 6·9sec.

CHAPTER VII.

LATITUDE.

The Altitude of the Pole is the Latitude of the Observer —Proofs — Proof for Meridian Altitudes — Rules — Examples of Meridian Altitudes of Sun and Stars, both above and below Poles—Artificial Horizon—Examination Questions.

90. The Altitude of the Pole is the Latitude of the Observer.

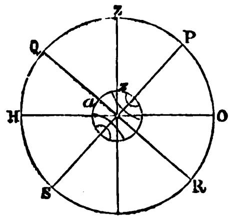


Let N W S E represent the sphere on the plane of the horizon, Z the zenith, P the north pole, W Q E the equator, and N P Z S the meridian.

The N P is the altitude of the pole above the horizon, and Z Q is the latitude of the place, for it is the distance of the observer's zenith from the equator.

Because the pole of a circle is 90° from every point of that circle, therefore $PQ = 90°$; for a similar reason $ZN = 90°$, $\therefore$ $PQ = ZN$. Omit the common part PZ from each, $\therefore$ $ZQ = PN$, or the altitude PN of the pole is equal to the latitude ZA of the observer. We might have said—

$$\because PQ = ZN$$
$$\therefore PZ + ZQ = PZ + PN$$
$$\therefore ZQ = PN$$



Second Proof.—Let SQPR represent the sphere on the plane of the meridian. Then Z is the zenith, HO the horizon, P the north pole, and QR the equator.

Let z be the position of a spectator. Then O P is the altitude of the pole above the horizon, and Q Z is the latitude of the observer at z, for $a\,z$ is the latitude, and arc $a\,z$ is equal to Q Z.

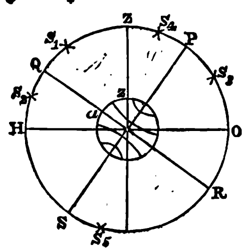
Now, Q P and Z O are both quadrants of circles;

$$\therefore \ QP = ZO$$
$$\therefore \ QZ + ZP = ZP + PO,$$

Take away the common part Z P $\therefore$ Q Z = P O, or, as before, the altitude P O of the pole P is equal to the latitude of the observer Q Z.

91. To show how the Altitude of any Celestial Object, when on the Meridian, will give the Latitude of the Observer.—We have just proved that the elevation of the pole is the latitude of the observer. It is an equally simple problem to find the latitude from the altitude of an object when on the meridian.

When an object is on the meridian, it is said to **culminate**; and its **culmination** is when it attains its highest point.



When an object crosses the meridian, we speak of it as its **transit** over the meridian.

Let this be a representation of the sphere on the plane of the meridian. Let P be the elevated pole of an observer at z, H O the horizon, Z the zenith of the observer, Q R the equator. Then Q Z or P O, just proved, is the latitude of the observer.

Let S_1 S_2 S_3 S_4 be four different positions of a celestial object.

The zenith distance is always equal to 90° − altitude.

(*a*) When an object is at S_2, its *declination* is S_2 Q, its altitude H S_2, and its zenith distance Z S_2.

$\therefore$ Latitude Z Q = S_2 Z − S_2 Q = zenith distance − declination.

that is, when the zenith is north, and the declination south, their difference is the latitude of the place.

(*b*) When an object is at S_1, its declination is Q S_1, its zenith distance S_1 Z, and its altitude H S_1.

$\therefore$ Latitude Z Q = S_1 Z + Q S_1 = zenith distance + declination.

that is, when the zenith is north, and the declination north, their sum is the latitude of the place.

(*c*) *When the zenith is south, and declination north, their difference is the latitude of the place.*

(*d*) *When the zenith is south, and declination south, their sum is the latitude of the place.*

From these four cases of meridian altitudes, we have these simple rules for finding the latitude. If the zenith and declination are both north or both south, add for the latitude; but if one be north, and the other south, their difference will give the latitude. When both are north or south, then the latitude is north or south; if one is north, and the other south, the latitude is of the same name as the greater.

(*e*) When an object is at S_3, it is said to be *below* the pole. Then its altitude is O S_3, its declination S_3 R, its zenith distance S_3 Z, and its polar distance S_3 P.

The polar distance is always equal to 90° − declination.

Q P = Z O, both being quadrants,

$\therefore$ Q Z + Z P = Z P + P O; take away the common part Z P,

$\therefore$ Q Z = P O,

$\therefore$ Latitude = P O = P S_3 + S_3 O = polar distance + altitude;

that is, when an object is *below* the pole, its polar distance added to its altitude will give the latitude of the place.

(*f*) When an object is at S_4, it is said to be *above* the

pole, then its altitude is O S_4, its declination is Q S_4, and polar distance S_4 P.

$$\text{Latitude} = \text{O P} = \text{O } S_4 - \text{P } S_4 = \text{altitude} - \text{polar distance};$$

that is, when an object is *above* the pole, its polar distance subtracted from its altitude will give the latitude of the place.

92. Latitude by a Circum-polar Star.—When the altitude of the same celestial object is taken on the meridian, both above and below the pole, then half the sum of the altitudes is the latitude of the place. This is proved thus—

Suppose S_3 and S_4 are denominated the inferior and superior positions of the same star; the polar distance is the same in each case.

$$\tfrac{1}{2} \text{ sum of alt.} = \tfrac{1}{2} (S_3 \text{ O} + S_4 \text{ O})$$
$$= \tfrac{1}{2} \left\{ (\text{P O} - S_3 \text{ P}) + (\text{P O} + S_4 \text{ P}) \right\}.$$

But since $S_3 \text{ P} = S_4 \text{ P}$

$$= \tfrac{1}{2} \left\{ 2 \text{ P O} \right\}$$
$$= \text{P O} = \text{latitude, or the elevation of the pole.}$$

Before illustrating the six preceding cases by examples, it will be perhaps well to make clear what is meant by the expression introduced into the questions on meridian altitudes—the sun bears south, or the zenith is north, or the opposite of both these cases.

If we say that the sun bears south, we mean that it is in the south part of the heavens; and, therefore, if we stand facing the sun, the zenith is north from it, for the north part of the horizon is behind us. Again, if the sun bears north, and we look towards it, the zenith bears south from it; or carrying the eye from the sun to the zenith, it would go towards the south point of the heavens. The student will find it particularly to his advantage to turn back to the last figure, and consider the object in the various positions in which it is there placed; if the object be south of the equator, let him consider where his zenith is from it north or south; and then he will perceive

that to obtain latitude, if the zenith is north and declination north, or both south, he must add the two to obtain latitude, and in all other cases subtract.

1. On July 9d., 1874, in longitude 51° 25′ 16″ E., the observed meridian altitude of the sun's lower limb was 49° 17′ 20″, the sun bearing S., the index error was −2′ 15″, and height of eye above the sea 18 ft.; what is the latitude?

	Greenwich Date. h. m. sec.	Lon. in Time.
Apparent time at place, July 9d.	0 0 0	51° 25′ 16″ E.
Longitude in time	3 25 41	4
Greenwich App. Time, July 8d.	20 34 19	6,0)20,5 41 4
		3h. 25m. 41 sec.

Declination.	H. D.	
July 8d. App. noon 22° 22′ 15″·1	−18·04	6,0)41
+1 1·8	3·43	6,0)25·7
22° 23′ 16″·9 N.	5412	·43
	7216	
	5412	
	61·8772	

To correct altitude and find latitude.

Obs. alt............	49° 17′ 20″
Error..............	−2 15
	49 15 5
Dip................	−4 8
	49 10 57
Semi-dia...........	15 46·1
	49 26 43·1
Ref.................	−49
	49 25 54·1
Para................	+6
True alt............	49 26 0·1
	90
Zen. dis.	40 34 0 N.
Dec.................	22 23 16·9 N.
Latitude	62 57 17 N.

This case is when the sun is at S_1 (fig. p. 82),

Explanation of this problem:—(1) The longitude is turned into time, and applied to the time at place to find the Greenwich date, according to the rules already laid down.

(2) The declination is taken from p. I. *Nautical Almanac* for apparent time, because the sun attains its meridian altitude at *noon, apparent time;* not noon, mean time. This declination is then corrected, as explained on p. 75. The hourly difference is marked –, which means that the declination is *decreasing,* and when marked + it means that it is *increasing*—so the two signs in this case are not used in their ordinary acceptation; for, as previously explained, if we go back, although the hourly difference may be marked plus, it will have to be subtracted, while, on the contrary, when it is marked minus and we go back in time, it will have to be added.

(3) The altitude is corrected by applying

(*a*) The index error;

(*b*) The dip, which is always subtracted;

(*c*) The semi-diameter, taken from p. II. of *Nautical Almanac;* it is *added* because we want the sun's centre: if upper limb had been observed, it would have been subtracted to find the centre;

(*d*) The refraction, from table book, which is always subtracted;

(*e*) The parallax, which is always added.

These five corrections give us the true geocentric altitude.

(*f*) The altitude is next subtracted from 90 to find the zenith distance. If the sun bears south, the zenith is *north;* it is therefore marked N; and the declination also being north, the two are added to give the latitude of the place, which is marked N, according to the rule on p. 84.

2. On February 14 day, 1874, in longitude 61° 25′ 36″ E., the observed meridian altitude of the sun's lower limb was 51° 17′ 45″,

zenith N., the index error was + 4′ 15″, height of eye 15 feet; required the latitude.

Greenwich Date.	h.	m.	sec.	Longitude in Time.
A. T. at place, Feb. 14day	0	0	0	61° 25′ 36″ E.
Lon. in time,...........	4	5	42·4	4
Greenwich A. T.,....13day	19	54	17·6	6,0)24,5 42 24
				4h. 5m. 42·4sc.

Declination.		m.	H.D.
Dec. Feb. 14day =	12° 58′ 2″·6 S.	5 = $\frac{1}{12}$	− 51·11
Diff..................	+ 3 29·27		4
Cor. dec.	13 1 31·87	sec.	204·44
		30 = $\frac{1}{10}$	4·26
		10 = $\frac{1}{3}$	·42
		2 = $\frac{1}{5}$	·14
			·02
		6,0	20,9·28
			3 29·28

Obs. alt..........	51° 17′ 45″
Index error.....	+ 4′ 15″
	51° 22′ 0″
Dip	− 3′ 46″·6
	51° 18′ 13″·4
Semi-diameter.	+ 16′ 13″·5
	51° 34′ 26″·9
Refraction......	− 45″
	51° 33′ 41″·9
Parallax.........	+ 5″
True alt.........	51° 33′ 46″·9
	90°
Zen. dis.........	38° 26′ 13″·1 N.
Declination.....	13° 1′ 31″·87 S.
Latitude.........	25° 24′ 41″·23 N.

This case is when the sun is at S_2

In this problem, although the declination is marked −, or decreasing, yet it is additive, the same as in the last problem, because we have corrected backwards for 4h. 5m. 42sec.

3. July 10 day, 1874, in longitude 35° 16′ 44″ W., the observed meridian altitude of the sun's lower limb was 45° 34′ 25″, sun bearing N., the index error was − 57″, height of eye 30 feet; required the latitude.

Greenwich Date.	h.	m.	sec.	Longitude in Time.
A. T. at place, July 10 day	0	0	0	35° 16′ 44″ W.
Lon. in time,..........	2	21	6·9	4
Greenwich A. T.,....10 day	2	21	6·9	6,0)14,1 6 56
				2h. 21m. 6·9sc.

Declination.		H. D.	
Dec. July 10 day = 22° 14′ 50″·5 N.		− 19″	6,0) 6·9
Diff................	− 44″·6	2·35	6,0)21·11
	22° 14′ 6″ N.	2115	·35
		235	
		44·65	

Obs. alt..........	45°	34′	25″
Index error.....			− 57″
	45°	33′	28″
Dip..............		− 5′	20″·5
	45°	28′	7″·5
Semi-diameter.		15′	46″·1
	45°	43′	53″·6
Refraction......			− 56″
	45°	42′	57″·6
Parallax.........			+ 6″
True alt.........	45°	43′	3″·6
	90°		
Zen. dist........	44°	16′	56″·4 S.
Declination.....	22°	14′	6″ N.
Latitude	22°	2′	50″·4 S.

As declination is north and zenith south, the difference is taken and the latitude is south, because the greater of the two is south.

93. Latitude by the Meridian Altitude of a Star.— This is a simpler problem, in theory, and enables an expert observer at sea to find his latitudes at any hour of the night, as there is always some bright star or other near the meridian. The declinations of the stars change

very slowly, so slowly that in the *Nautical Almanac* it is only considered necessary to give the declination of the principal stars for every ten days. It is unnecessary to find a Greenwich date, and, as stars have no semi-diameter or parallax, we simply *correct* for index error, refraction, and dip, then find the zenith distance and apply the declination. The result will be the latitude of the ship or observer.

4. On Aug. 21 day, 1874, the observed meridian altitude of Arcturus was 71° 17′ 30″, zenith N., index error − 2′ 25″, height of eye 25 feet; required the latitude.

Obs. alt.	71°	17′	30″	
Index error.....		−2	25	
	71	15	5	
Dip.		−4	52·5	
	71	10	12·5	
Ref.			−19	
True alt.	71	9	53·5	
	90			
Zen. dis..........	18	50	6·5	N.
Dec..............	19	50	21·4	N.
Latitude.........	38	40	27·9	N.

5. On September 8 day 1874, the observed meridian altitude of α Serpentis, bearing N., was 39° 17′ 20″, index error + 3′ 17″, height of eye 12 feet; find the latitude.

Obs. alt.	39°	17′	20″	
Error............		+3	17	
	39	20	37	
Dip...............		−3	22	
	39	17	15	
Ref.		−1	9	
True alt..........	39	16	6	
	90			
Zen. dis.	50	43	54	S.
Dec................	6	49	21·2	N.
Latitude	43	54	32·8	S.

6. January 1, 1874, the observed meridian altitude of β Hydri above the south pole was 49° 16′ 40″, the index correction was − 3′ 45″, height of eye 16 feet; required the latitude.

Obs. alt. ...	49°	16′	40″
Index error		−3	45
	49	12	55
Dip..........		−3	54
	49	9	1
Ref...........			−49
True alt.....	49	8	12
Polar dis....	12	1	42·7
Latitude....	37	6	29·3 S.

Dec........	77°	58′	17″·3 S.
	90		
Polar dis.	12°	1′	42″·7

This case is when the star is at S_5.

7. January 10 day 1874, the observed meridian altitude of α Cassiopeiæ *below* the north pole was 37° 18′ 30″, the index correction was + 1′ 25″, and height of eye 14 feet; required the latitude.

Obs. alt. ...	37°	18′	30″
Index error		+1	25
	37	19	55
Dip..........		−3	38·8
	37	16	16·2
Ref.		−1	15
True alt. ...	37	15	1·2
Polar dis....	34	9	2·5
Latitude ...	71	24	3·7 N.

Dec........	55°	50′	57″·5 N.
	90		
Polar dis.	34°	9′	2″·5

This case is when the star is at S_8.

94. Latitude by Observations above and below the Pole.

8. January 31 day 1874, the observed altitude of α Cassiopeiæ on the meridian below the north pole was 17° 16′ 40″, and above the pole the same night it was 85° 32′ 40″, index correction in each case was − 3′ 18″, height of eye 9 feet; what was the latitude?

Obs. alt.	17°	16′	40″		85°	32′	0″
Error ...		−3	18			−3	18
	17	13	22		85	28	42
Dip		−2	55·5			−2	55·5
	17	10	26·5		85	25	46·5
Ref.		−3	3		17	7	23·5
	17	7	23·5	2)	102	33	10
				Latitude....	51°	16′	35″ N.

One proof of this last process has been already given.

The reason for adding these two altitudes and taking the mean to find the latitude, is very evident from the figure preceding. Suppose S_3 to be a circumpolar star, as it moves round the pole, the circle it describes will be always at the same distance from the pole, because its pole is P, hence when at S_4 we see that S_4 and S_3 being the same distance from P, the latitude or elevation of the pole is $\frac{1}{2}(O S_3 + O S_4)$.

$$= \tfrac{1}{2}(O S_3 + O S_3 + P S_3 + P S_4) = \tfrac{1}{2}(2 O S_3 + 2 P S_3)$$
$$= O S_3 + P S_3 = O P.$$

$\therefore$ half the sum of the meridian altitudes of a celestial object above and below the pole is the latitude of the observer.

9. July 10 day 1874, the observed meridian altitude of β Chamæleontis in an artificial horizon below the south pole was 86° 25′ 54″, and above the pole 131° 55′ 24″, index correction −1′ 18″; what was the latitude?

To correct altitudes and find latitude.

Obs. alt.....	86° 25′ 54″	131° 55′ 24″
Index error	−1 18	−1 18
	2)86 24 36	2)131 54 6
	43 12 18	65 57 3
Ref.........	−1 1	−25
True alt.....	43 11 17	65 56 38
	65 56 38	
	2)109 7 55	
Latitude	54 33 57·5 S.	

95. Artificial Horizon.—It will be observed that a new term, "artificial horizon," is here introduced. It is an instrument consisting essentially of a small sheet of mercury, in which, when the sea horizon is obscured, the altitudes of objects are taken, by bringing with the sextant the true and reflected images together in the mercury. When an altitude is taken in an artificial horizon, the angle measured is double the real altitude. Hence we apply the index error, and take half the altitude for the observed altitude. To an altitude taken under these circumstances no dip is applied.

EXERCISES FROM EXAMINATION PAPERS, THE DATES BEING ALTERED TO 1874.

10. August 16, 1874, in longitude 72° 41′ W., the observed meridian altitude of the sun's L. L. was 49° 16′ 40″ (zenith N. of the sun), the index correction was – 2′ 10″, height of eye 30 feet; required the latitude.

Explain the reasons for the corrections you have applied to your observed altitude (1870).

Aug. 16 day, Declination = 13° 43′ 51″ N., H. D. – 47″·47.
,, Semi-diam. = 15′ 50″·2.

Ans. Lat. 54° 15′ 44″·3 N.

11. June 14, 1874, in longitude 30° 30′ E., the observed meridian altitude of the sun's lower limb was 59° 45′ 40″ (zenith N. of sun), index error – 3′ 50″, and the height of the eye above the sea was 19 feet; required the latitude (1866).

June 14 day, Declination = 23° 16′ 57″·4 N., H. D. + 7″·36.
,, Semi-diam. = 15′ 46″·7.

Ans. Lat. 53° 24′ 19″·6 N.

12. January 7, 1874, the observed meridian altitude of the sun's L. L. (zenith S. of sun), was 48° 13′ 25″, index error – 1′ 20″, and the height of the eye above the sea was 17 feet; required the latitude (1866), longitude 16° 18′ 25″ W.

Jan. 7 day, Declination = 22° 21′ 48″·5 S., H. D. – 19″·34.
,, Semi-diam. = 16′ 18″·1.

Ans. Lat. 63° 57′ 50″ S.

13. May 1, 1874, the observed meridian altitude of sun's L. L. was 75° 10′ 50″ (zenith N. of the sun), index correction – 5′ 50″, and height of the eye above the sea 30 feet; required the latitude (1866), longitude 50° 30′ W.

May 1 day, Declination 15° 6′ 37″·7, H. D. + 45″·29.
,, Semi-diam. 15′ 54″.

Ans. Lat. 29° 53′ 49″·6 N.

14. August 9, 1874, in lon. 37° 30′ W., the meridian altitude of the sun's L.L., was 53° 20′ 40″ (zenith N. of the sun), the index correction was + 4′ 20″, and the height of the eye above the sea was 19 feet: required the altitude (1863).

Aug. 9d. Dec. = 15° 51′ 8″·5 N., H.D. – 43″·33.
,, Semi-dia. 15′ 49″.

Ans. Latitude 54° 13′ 26″·3 N.

15. February 19, 1874, the observed meridian altitude of α Arietis, was 50° 25′ 10″ (zenith S. of star), the index error was + 3′ 10″, and the height of the eye was 12 feet: required the latitude (1866).

February 20, Dec. α Arietis = 22° 51′ 58″·3 N.

Ans. 16° 43′ 50″·7 S.

State the rule for finding the latitude of a place by observed

meridian altitude of a heavenly body, and illustrate by a diagram (1860).

16. January 7, 1874, the observed meridian altitude of the sun's L.L. (zenith S. of sun), was 48° 13′ 25″, index error − 1′ 20″, and the height of the eye above the sea was 17 feet: required the latitude (1861), Lon. 55° 17′ 20″ W.

January 7d. Dec. from N.A. = 22° 21′ 48″·5 S.
H.D. − 19·34, semi-diameter 16′ 18″·1

Ans. Latitude 63° 56′ 59″ S.

17. The altitude of the pole above the horizon is equal to the latitude of the place: prove this (1868).

Prove the rule for calculating the correction called "the reduction of the moon's horizontal parallax." Prove the rule for calculating the parallax in altitude from the horizontal parallax.

18. What is meant by correction for parallax? How is it applied (1869)?

19. Find the rule for finding the latitude by a meridian altitude of a heavenly body above the pole, and draw diagrams for the following cases:—(1) Latitude of the place N., declination of the body N., body S. of zenith; (2) latitude N., declination N., body N. of zenith; (3) latitude N., declination S.; (4) latitude S., declination S., body N. of zenith; (5) latitude S., declination S., body S. of zenith; (6) latitude S., declination N. (1868).

20. Prove the rule for finding the latitude by means of an observed altitude of a heavenly body, when the zenith distance is N., and the declination S.

21. February 18, 1874, at a place in longitude 24° W., the meridian altitude of sun's upper limb is observed to be 18° 14′, zenith N. of sun, index error + 2′ 30″, and height of eye 18 feet: required the latitude (1862.)

February 18d., Dec. from N.A. = 11° 34′ 39″·8 S.
H.D. − 53″·06, semi-diameter 16′ 12″·6.

Ans. Latitude 69° 0′ 51″·4 N.

22. September 28, 1874, the observed meridian altitude of α Cygni was 63° 5′ 30″, zenith N. of star, index error − 1′ 55″, the height of eye 16 feet: required the latitude (1862).

September 28, Dec. of α Cygni = 44° 50′ 8″·8 N.

Ans. Latitude 71° 50′ 7″·8 N.

23. Show how to find the latitude by the observed meridian altitude of the sun, moon, or a star (1863).

24. April 20, 1874, the observed meridian altitude of Aldebaran was 53° 40′ 10″ (zenith N. of star), index correction was + 1′ 10″, and the height of the eye above the sea 16 feet: required the latitude (1864).

April 20d. Dec. of Aldebaran is 16° 15′ 18″ N.

Ans. Latitude 52° 38′ 34″ N.

25. June 27, 1874, the observed meridian altitude of Regulus

(zenith S. of star), was 59° 18′ 25″, the index correction − 5′ 50″, the height of the eye above the sea 30 feet: required the latitude (1866).

June 27d. Dec. of Regulus = 12° 34′ 58″·3 N.

Ans. Latitude 18° 18′ 21″·2 S.

26. May 20, 1874, the observed meridian altitude of α Cruis, was 74° 13′ 20″ (zenith N. of star), index error + 1′ 20″, the height of the eye 12 feet: required the latitude (1867).

May 20d. Dec. of α Cruis = 62° 24′ 18″·9 S.

Ans. Latitude 46° 35′ 22″·9 S.

27. July 4, in longitude 36° 50′ W. the observed meridian altitude of sun's L.L. was 36° 16′ 20″ (zenith N. of sun), index error − 3′ 55″, and the height of the eye 16 feet: required the latitude (1865).

July 4d. Dec. 22° 53′ 26″·6 N., H.D. = − 13″·63, semi-dia. 15′ 46″.

Ans. Latitude 76° 21′ 57″·6 N.

To find the altitude of an object when on the meridian.—In finding latitude it was proved that the latitude of the observer is always equal, when the celestial object is culminating, to the difference or sum of the zenith distance and declination, according as they are marked north or south.

(1) N. latitude = zenith dis. (N.) + dec. (N.)
∴ Zen. dis. = N. lat. − dec. (N.)
and Alt. = 90° − zen. dis.

(2) N. latitude = zen. dis. (N.) − dec. (S.)
∴ Zen. dis. = N. lat. + dec. (S.)
Alt. = 90° − zen. dis.

(3) N. latitude = dec. (N.) − zen. dis. (S.)
∴ Zen. dis. = dec. N. − N. lat.
Alt. = 90° − zen. dis.

We may treat south latitude the same, but the student must accustom himself to all these changes, and by varying the question, he can find the altitude of an object at its transit at any place at noon.

1. The declination of the sun is 16° 18′ 12″ S. and the latitude of an observer is 48° 17′ 21″ N.; what is the altitude of the sun at noon? *Ans.* 25° 24′ 27″.

2. The declination of the sun is 21° 17′ 18″ S. and the latitude of the observer is 45° 16′ 19″ N.; find the altitude of the sun when on the observer's meridian. *Ans.* Alt. 23° 26′ 23″.

3. Find at what altitude, at a place in longitude 85° 17′ 21″ E. the sun passes the meridian on June 18d., the observer being in latitude 60° 17′ 24″ N.

Greenwich date.

		h.	m.	sec.
A. T. at place, June 18d.,		0	0	0
		5	41	9·4
App. G. date,	17d.,	18	18	50·6

Longitude.

85° 17′ 21″ E.
4
6,0)34,1 9 24
5h. 41m. 9·4sec.

Declination.

18d.	= 23° 25′ 26″·3
	18″·4
Cor. dec.	23° 25′ 8″ N.

H.D.

30 = ½	+ 3·24
	5
	16·20
10 = ⅓	1·62
1 = $\frac{1}{10}$	·54
	·05
	18·41

9·4sec. are neglected.

N. Lat. = zen. dis. (N.) + Dec. (N.)
∴ Zen. dis. = N. lat. − Dec. (N.)

Zenith distance.

Lat.	60° 17′ 24″ N.
Dec.	23 25 8 N.
Zen. dis.	36 52 16 N.
	90
Alt. of sun,	53° 7′ 44″ *Ans.*

4. Find at what altitude the sun will pass the meridian of a place in lat. 51° 16′ 17″ N., lon. 8° 10′ W., on June 1d. 1874.
Dec. June 1d., = 22° 4′ 34″·6
H.D. = +20″·32 *Ans.* Alt. 60° 41′ 28″.

5. Find at what altitude the sun will pass the meridian of a place in lat. 45° 20′ N., lon. 51° 4′ 20″ E., on May 4d. 1872.
Dec. May 4d. = 15° 59′ 50″·2
H.D. = +43″·37 *Ans.* Alt. 60° 37′ 23″.

6. Find at what altitude the sun will pass the meridian of a place in lat. 45° 16′ N., lon. 1° 50′ E., on Feb. 10d. 1874.
Dec. Feb. 10d. = 14° 18′ 3″·4 S.
H. D. = −48″·87 *Ans.* Alt. 30° 17′ 48″.

7. Find at what altitude the sun will pass the meridian of a place in lat. 10° 16′ S., lon. 51° W., on June 20d. 1874.
Dec. June 20d. = 23° 27′ 12″ N.
H. D. = +1″·16 *Ans.* Alt. 56° 16′ 44″.

CHAPTER VIII.

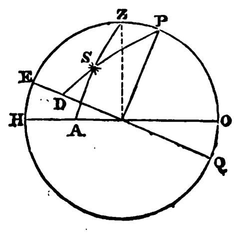
LONGITUDE BY CHRONOMETER.

The Condition of the Problem—Simple Illustration—the Elements of the Problem — Borda's Proof adapted to Logarithmic Computation—Examples—Sun Chronometer—Star Chronometer—Rules—Examples—Examination Questions.

96. Longitude is found at sea by many methods, but the simplest and the one most generally adopted will now be explained. When a master takes his vessel to sea, he secures a good chronometer, whose error and rate are known. Knowing the error with the daily rate of gain or loss, he can always tell the exact mean time at Greenwich. A watch cannot keep apparent time, because, as has already been explained, apparent days differ in length, being sometimes longer and sometimes shorter than at other times; in fact, what is called the equation of time, which must be added to or taken from mean time to find apparent time, amounts at one time, Nov. 3d., 1874, to 16m. 19·21sec. to be added to, and at another, Feb. 11 d., to 14m. 29·18sec. to be subtracted from, mean time to find apparent time. Hence a watch, if it indicates anything, must necessarily indicate Greenwich mean time (G.M.T). The problem of finding longitude at sea simply resolves itself into this: if we can find the time at place, we can carry with us the Greenwich time, and the difference between these two times will be the longitude in time. Every altitude of the sun corresponds to a certain time of the day, before or after noon. Hence, if we obtain the altitude, we can find the time corresponding to this altitude; this is simply the problem now to be solved.

To find longitude is the most important problem in Nautical Astronomy. The finding of longitude depends upon calculating the hour angle and mean time at place; then, if we can, from a good chronometer, obtain the correct Greenwich mean time, the difference between the two times is the longitude in time. The problem of finding longitude by chronometer depends upon the latitude of the observer, the altitude of the celestial object, and the declination of the sun. From these elements the apparent time at ship is found; the above, or their complements, form the three sides of a spherical triangle, from which the angles can be calculated. The angle opposite the zenith distance, the complement of the altitude, is the apparent time at place in arc; this being found and the equation of time applied to it gives the mean time at ship, and the difference between the mean time at ship and the Greenwich time is the longitude in time; for instance, let it be supposed that we have worked out a problem, and find that when it is two o'clock in the afternoon at Greenwich it is eleven in the morning at ship, the difference in time is *three* hours or 45°, *i.e.*, the ship is 45° to the west of Greenwich; had it, on the other hand, been one o'clock in the afternoon at Greenwich and four o'clock at ship, the difference, three hours or 45°, is the longitude east of Greenwich.

Proof of Rule for finding longitude by chronometer. We will give Borda's proof and rule, than which no better exists. It is found in every work on Nautical Astronomy.



Let this be a representation of the sphere of the heavens. P be the North Pole, Z the zenith, E Q the equator, H O the rational horizon of the observer, P O or E Z the latitude of the observer $= l$, Z P is the co-latitude $= (90 - l)$. Let S be a celestial object whose altitude S A is a.

Then S Z is the zenith distance $= 90 - a$

Let the great circle passing through P and S be continued till it cuts the equator in D, then

D S is the declination of S $= d$.

S P is the polar distance of S $= p = (90 - d)$.

Then the hour angle is the angle Z P S $= h$.

From the triangle Z P S we have, by the ordinary rules of spherical trigonometry,

$$\text{Cos. } ZPS = \frac{\text{Cos. } ZS - \text{Cos. } ZP \text{ Cos. } PS.}{\text{Sin. } ZP \text{ Sin. } PS.}$$

Hence by substituting the terms given above for these, we get

$$\text{Cos. } h = \frac{\text{Cos. } (90 - a) - \text{Cos. } (90 - l) \text{ Cos. } p,}{\text{Sin. } (90 - l) \text{ Sin. } p.}$$

$$= \frac{\text{Sin. } a - \text{Sin. } l \text{ Cos. } p.}{\text{Cos. } l \text{ Sin. } p.}$$

Subtracting each side from unity,

$$1 - \text{Cos. } h = 1 - \frac{\text{Sin. } a - \text{Sin. } l \text{ Cos. } p.}{\text{Cos. } l \text{ Sin. } p.}$$

$$= \frac{\text{Cos. } l \text{ Sin. } p + \text{Sin. } l \text{ Cos. } p - \text{Sin. } a.}{\text{Cos. } l \text{ Sin. } p.}$$

$$= \frac{\text{Sin. } (p + l) - \text{Sin. } a.}{\text{Cos. } l \text{ Sin. } p.}$$

Remembering that $\text{Cos. } A = 1 - 2 \text{ Sin.}^2 \frac{A}{2}$ and that

$$\text{Sin. } A - \text{Sin. } B = 2 \text{ Cos. } \frac{A + B}{2} \text{ Sin. } \frac{A - B}{2}, \text{ we have}$$

$$2 \text{ Sin.}^2 \frac{h}{2} = \frac{2 \text{ Cos. } \frac{p + l + a}{2} \text{ Sin. } \frac{p + l - a}{2}}{\text{Cos. } l \text{ Sin. } p.}$$

$$\therefore \text{ Sin. } \frac{h}{2} = \left\{ \text{Cos. } \frac{p + l + a}{2} \text{ Sin. } \frac{p + l - a}{2} \text{ Sec. } l \text{ Cosec. } p \right\}^{\frac{1}{2}}$$

When we let $S = \frac{p + l + a}{2}$ we have, by taking the logarithm of both sides the equation,

$$\text{log. Sin. } \frac{h}{2} = \frac{1}{2} \left\{ \text{log Cos. } S + \text{log Sin. } (S - a) + \text{log Cosec. } p + \text{log Sec. } l \right\}$$

Rules for finding longitude by chronometer—

(*a*) Find the Greenwich date, as in the preceding examples, correcting it for the rate of the chronometer.

(*b*) Correct the declination, and find the polar distance :—

The *polar distance* is the distance of the celestial object from the nearest pole to the spectator. If the declination is N. and latitude S., or *vice versa*, add 90° to the declination for the polar distance; but if both declination and latitude are of the same name, subtract from 90° for the polar distance.

(*c*) Correct the equation of time: both declination and equation of time must be taken from II. page of *Nautical Almanac*, but the equation of time must be applied according to the directions on page I.

(*d*) Correct the altitude.

(*e*) Add together altitude, latitude, and polar distance, and divide by *two*. Subtract the altitude from the half sum, and call the remainder the difference.

Take out the logarithms Sec. lat., Cosec. pol. dis., Cos. half sum, and Sin. difference; add these four logarithms together, and take half their sum, then the sine of this half sum is half the hour angle in arc.

(*f*) Having found the hour angle, reduce it to time; and if the object be east of the meridian,* then subtract this from 24 hours. This is the apparent time at place. Apply the equation of time, and you have mean time at ship.

(*g*) Now find the difference between the mean time at ship and the Greenwich date: this is the longitude in time.

(*h*) The longitude is E. if the Greenwich date is earlier in the day than the ship time, and W. if it be later.

An example will now be worked out, applying the

* The sun is E. of the meridian in the morning, and W. in the afternoon.

rules given above, by which the longitude may be found by chronometer under any circumstances.

1. June 12d. P.M. at ship in latitude 34° 10′ S., when a chronometer showed 1h. 20m. 10sec. P.M. at Greenwich, the observed altitude of the sun's lower limb was 16° 49′ 37″, index error − 1′ 17″, height of eye 16 feet; required the longitude.

On May 12d. at noon the chronometer was slow 35m. 30sec. and losing daily 2 seconds.

Greenwich Time from chronometer.	h.	m.	sec.	Rate. sec.
June 12d.	1	20	10	2
Slow	0	35	30	30
	1	55	40	60
Loss	0	1	0	
G. M. T. June 12d.	1h.	56m.	40sec.	

Declination.	H. D.	Eq. of Time.	H.D.
June 12d. = 23° 10′ 14″·8 N.	+9·41″	0′ 31″·24	− ·515
+18″·2	1·94	− 0″·99	1·94
23° 10′ 33″	3764	− 0′ 30″·3	2060
90	8469		4635
Polar dis. = 113° 10′ 33″	941		515
	18·2554		·99910

To correct the altitude—

	°	′	″
Obs. altitude	16°	49′	37″
Index error		−1	17
	16	48	20
Dip		−3	54
	16	44	26
Semi-diameter		+15	47
	17	0	13
Refraction		−3	1
	16	57	12
Parallax			+8
	16	57	20

To calculate the hour angle and mean time at place—

$a =$	16° 57′ 20″		
$l =$	34° 10′ 0″	Sec.	10·082281
$p =$	113° 10′ 33″	Cosec.	10·036540
2)	164° 17′ 53″		
	82° 8′ 56″	Cos.	9·135451
	16° 57′ 20″		
	65° 11′ 36″	Sin.	9·957955
			2)19·212227
Half-hour angle,	23° 48′ 46″	Sin.	9·606113
	2		
Hour angle,...	47° 37′ 32″		
	4		
6,0)	19,0° 30′ 8″		

	h.	m.	sec.
Apparent time at place,...........	3h.	10m.	30·1sec.
Equation of time, ...	−0		30·3
Mean time at place, June 12d....	3	9	59·8
M. T. at Greenwich, June 12d....	1	56	40
Difference, ...	1	13	19·8
	60		
	4)73	19	48
Longitude...........................	18°	19′	57″ E.

2. Nov. 10 day A.M. at ship (Port Louis in Mauritius), in latitude 20° 10′ N., when a chronometer showed 6h. 54m. A.M. of same day at Greenwich, the observed altitude of the sun's lower limb was 46° 35′ 30″, index error +2′ 17″, height of eye 19 feet; required the longitude.

On August 14 day the chronometer was 25m. 15sec. fast, and gaining 2·4 seconds per day.

Greenwich Date.	h.	m.	sec.	Rate.
				88 days
Chronometer Nov. 9 day......	18	54	0	2·4
Fast.......................		25	15	352
	18	28	45	176
Rate.......................	−	3	31	6,0)21,1·2
Greenwich M. T. Nov. 9 day.	18	25	14	3·31

Declination.		H. D.
Nov. 10 day	17° 11′ 43″·2 S.	+42″·14
	3′ 55″	5·58
Cor. Dec.	17° 7′ 48″·2	33712
	90	21070
Pol. dis.	107° 7′ 48″	21070
		6,0)23,5·1412
		3′ 55″

Equation of Time.	H. D.
m. sec.	
15 56·08	−·261
+1·45	5·58
−15 57·53	2088
	1305
	1305
	1·45638

To correct the altitude—

Obs. alt.	46° 35′ 30″
Index error	+2′ 17″
	46° 37′ 47″
Dip	−4′ 15″
	46° 33′ 32″
Semi-diameter	+16′ 11″·8
	46° 49′ 43″·8
Refraction	−54″
	46° 48′ 49″·8
Parallax	+6″
True altitude	46° 48′ 56″

To calculate hour angle and mean time at place—

	$a=$ 46° 48′ 56″		
	$l=$ 20° 10′ 0″	Sec.	10·027470
	$p=$107° 7′ 48″	Cosec.	10·019706
	2)174° 6′ 44″		
	87° 3′ 22″	Cos.	8·710591
	46° 48′ 56″		
	40° 14′ 26″	Sin.	9·810229
			2)18·568002
Half-hour angle arc	11° 5′ 15″·3	Sin.	9·284001
	2		
Hour angle arc	22° 10′ 30″·6		
	4		
	6,0)8,8° 42′ 2″		
[illegible]r angle, time, E.	1h. 28m. 42sec.		

	h. m. sec.
Hour angle time, E............	1 28 42
	24
Hour angle time, W............	22 31 18
Equation of time, ...	−15 57·53
Mean time at place Nov. 9d.	22 15 20·47
M. T. at Greenwich Nov. 9d.	18 25 14
	3 50 6·47
	60
	4)230 6 28
Longitude	57° 31′ 37″ E.

3. February 9 day 1874, about 10 minutes past 9 in the evening at Plymouth, in latitude 50° 22′ 25″ N., when the chronometer showed 9h. 38m. 1sec. the observed altitude of ι Hydræ was 38° 55′ 20″ E. of the meridian, index error − 4′ 17″, height of eye 54 feet; required the longitude.

On January 1 day the chronometer was 10m. 12sec. too fast, and was gaining $1\frac{1}{2}$ sec. per day.

Greenwich Date.

	h. m. sec.
Chronometer............	9 38 1
Fast.......................	−10 12
	9 27 49
Gain	−1 0
Green. M.T. Feb. 9d.	9 26 49

Rate.

40
$1\frac{1}{2}$
40
20
60 sec.

	h. m. sec.
R.A. of ι Hydræ =	8 40 7·21
Dec. ,,	6° 52′ 47″ N.
Polar dis. ,,	83° 7′ 13″

R.A. of Mean Sun.

		h. m. sec.
Sidereal time,		21 17 37·8
Accel.	9h.	1 28·7083
	26m.	4·2711
	49sec.	·1342
		21 19 10·9136

To correct the altitude—

Obs. altitude.......	38° 55′ 20″
Index error.........	−4′ 17″
	38° 51′ 3″
Dip..................	−7′ 10″
	38° 43′ 53″
Refraction..........	1′ 12″
True altitude......	38° 42′ 41″

To calculate hour angle and mean time at place—

$a=$	38° 42′ 41″		
$l=$	50° 22′ 25″	Sec.	10·195329
$p=$	83° 7′ 13″	Cosec.	10·003139
	2)172° 12′ 19″		
	86° 6′ 9″	Cos.	8·832330
	38° 42′ 41″		
	47° 23′ 28″	Sin.	9·866874
			2)18·897672
Half-hour angle,	16° 19′ 29″	Sin.	9·448836
	2		
Hour angle arc...	32° 38′ 58″		
	4		
	6,0)13,0° 35′ 52″		
Hour angle time, E.	2h. 10m. 35·8sec.		
	24		
Hour angle time, W.	21 49 24·2		
R.A. of star..................	8 40 7·21		
	30 29 31·41		
R.A. of mean sun............	21 19 10·91		
Time at Plymouth, Feb. 9d.	9 10 20·5		
Time at Greenwich, Feb. 9d.	9 26 49		
	4)16 28·5		
Longitude......................	4° 7′ 7″·5 W.		

Remarks on the three preceding examples :—

(1) As the declination is N. and the latitude S., the distance of the nearest pole to the spectator from the sun is found by adding 90° to the declination. In the next example declination is S. and latitude N., hence the same rule applies. In taking out Cosec. 113° 10′ 32″, either take out Sec. 23° 10′ 32″ or Cosec. 66° 49′ 28″. The longitude is E., because at Greenwich it is earlier in the day.

(2) In this it will be noticed that the declination is corrected backwards, hence although the difference is marked + it is subtracted. The Cosec. of 107° 7′ 48″ is the same as Sec. 17° 7′ 48″. The observation for finding longitude was taken in the morning, hence the hour angle is subtracted from 24 hours.

(3) This is a star chronometer; the Greenwich date is found as usual. In stars the declination requires no correction, as it changes but slowly. It is an eastern hour angle. The time at Greenwich is later in the day than at ship, hence the longitude is W.

All the following examples are taken from examination papers; properly, to suit 1874, the altitudes as given should be re-calculated, but for obvious reasons they are given precisely as the examiner gave them. In the case of some a different latitude or altitude ought to be given to keep them off the land; as they stand they are not quite possible questions, but with that we have nothing to do; they are mere exercises to test whether the student under given circumstances can apply the rules he has learnt. Again, in some of the previous illustrations we have given examples which some over nice people would call "impossible questions," because if applied to a ship they would bring it on the land; but they are not applied to a ship, but to any observer (we have merely supposed he can get a distinct horizon), and we have used them because they illustrate all points under consideration much better than ordinary examples.

Although we cannot be too precise and correct in Nautical Astronomy, it is useless conjuring up such needless difficulties as above referred to. Before proceeding we again call attention to one or two points.

4. In example 5, following, the mean time at place and the longitude *nearly* or by account are given; from these we get the Greenwich date thus :—

Greenwich date nearly.	h.	m.	sec.	Longitude.
Ap. 30d.	19	0	0	29° 30′ W.
Long. W.	1	58	0	4
Ap. 30d.	20	58	0	6,0)11,8 0
				1h. 0m. 58sec.

Now we learn from this that the 8 o'clock shown by the chronometer is 20 hours at Greenwich, and we have

Greenwich date by Chronometer.	h.	m.	sec.	Rate.
April 30d.	20	24	29	2·2
Slow		34	50	27 days elapsed.
	20	59	19	154
Rate loss			59·4	44
April 30d.	21	0	18·4	59·4

The equation of time is marked – or + to be subtracted or added. This is one of those points in which the student cannot always fully realize the exact state of the case without using the *Nautical Almanac.* Although all the elements for working each question accompany it, the student, from the commencement, will be wiser to entirely discard them, and use the *Nautical Almanac*, which he *must* do in any examination. We were going to speak of equation of time: the equation of time is taken from the *second* page of the month, because we are working with *mean* time; but when our hour angle comes out it is apparent time, therefore the student must be guided by the *first* page as to what he must do with the equation of time, where it states whether it is to be added to or subtracted from apparent time (to give mean time).

97. **The Longitude by Account** is the longitude as given by the ship's dead reckoning.

98. **The Hour Angle is E. or W.**—If it be the sun it is east if the time be A.M. at ship, west if P.M.; but as regards a star or any other celestial object, at the time of taking the altitude, the observer must notice whether it be east or west of the meridian.

5. May 1, 1874, in latitude 29° 58′ N., longitude by account 29° 30′ W., at 7 A.M. mean time at ship, the observed altitude of sun's L. L. was 21° 10′ 11″, index corr. – 5′ 50″, the height of the eye above the sea 30 feet, at the same time a chronometer showed 8h. 24m. 29sec.; April 3 the chronometer was slow on G. M. T. 34m. 50sec., and its daily rate was 2·2sec. losing; required the longitude (1868).

May 1d.	Dec. = 15° 6′ 40″ N.	H.D. + 45·29	
	m. sec.		
,,	Eq. of time – 3 2·81	H.D. + ·318	
,,	Semi-dia. 15 54		*Ans.* Longitude 29° 45′ 3″ W.

6. October 30, 1874, P.M., in latitude 42° 7′ N., the chronometer showed 6h. 10m. 6sec. (it being October 31, A.M., civil time at Greenwich) when the observed altitude of sun's L.L. was 16°46′10″, index error – 3′ 55″, and the height of the eye 17 feet; required the longitude.

On September 27, at noon, the chronometer was too slow

on Greenwich mean time 4m. 18·4sec., and gaining 5·2sec. daily (1871).

Oct. 31d. Dec. 14° 9′ 35″·2 H.D. +48″·64

		m. sec.	
,,	Eq. of time	−16 16·57	H.D. + ·088
,,	Semi-dia.	16 9·5	

Ans. Longitude 134° 0′ 30″ E.

7. March 4, 1874, at 2h. 42m. P.M. (mean time nearly), in latitude 48° 48′ N. and longitude by account 41° W., when a chronometer showed 6h. 13m. 4sec., the observed altitude of the sun's L. L. was 26° 16′ 50″, index correction + 3′ 20″, height of eye 19 feet; on February 2, the chronometer was fast 46m. 10·5sec. on G. M. T., and its rate is 2·7sec. gaining; required longitude (1870).

March 4d. Dec.	= 6° 23′ 7″·4 S.	H.D. −57″·72
	m. secs.	
Eq. of time	= + 11 54·55	H.D. − ·561
Semi-diameter	= 16′ 9″·4	

Ans. Longitude 41° 38′ W.

8. January 2, 1874, at 5h. 58m. A.M. mean time nearly, in latitude 51° 15′ N., longitude by account 65° 40′ W., when a chronometer showed 10h. 24m. 50sec., the observed altitude of Pollux (W. of meridian) was 29° 54′ 40″, the index correction + 5′ 55″, and the height of the eye above the sea 14 feet; required the true longitude.

On December 15 the chronometer was fast on Greenwich mean time 5m. 20·5sec., and its daily rate is 3·5sec. gaining (1864).

Dec. of Pollux, ... 28° 19′ 46″·2 N.
R.A. of Pollux, ... 7h. 37m. 37·01sec.
Sidereal time, Jan. 1d. = 18h. 43m. 52·06sec.

Ans. Longitude 65° 36′ W.

9. October 11, 1874, at 3h. 15m. A.M., in latitude 41° 10′ S., and longitude by account 8° 30′ E., the observed altitude of α Arietis, W. of the meridian, was 16° 49′ 50″, when the chronometer showed 1h. 35m. 45sec., the index error was − 3′ 25″, and the height of the eye 18 feet; required the longitude.

On Oct. 1d. the chronometer was 1h. 6m. 10sec. slow on mean time, and daily losing ·5sec.

Dec. of α Arietis, ... 22° 52′ 16″ N.
R.A. ,, ... 2h. 0m. 7·55sec.
Sidereal time, Oct. 10d. = 13h. 15m. 40·82sec.

Ans. 8° 33′ 57″·5 E.

10. May 11, 1874, P.M., in latitude 48° 30′ N., the chronometer showed 6h. 5min. 40sec. (it being May 12 A.M. at Greenwich), when the observed altitude of sun's L. L. was 37° 22′ 10″, the

index correction was +3′ 10″, and the height of the eye above the sea was 18 feet; required the longitude.

On April 19 at noon, the chronometer was fast on Greenwich mean time 20m. 10·8sec., and its daily rate was 3·5sec. losing (1864).

May 12d. Dec.......... = 18° 9′ 49″·4 N., H.D. +37″·71
,, Eq. of time = −3m. 52″·31sec., H.D. − ·053
Semi-diameter......... = 15′ 51″·7.

Ans. 145° 30′ 15″ E.

11. May 15, 1874, at 8h. 33m. A.M., in latitude 40° 40′ N., longitude by account 179° 55′ W., the observed altitude of the sun's L. L. was 41° 30′ 30″, when the chronometer showed 8h. 34m. 14sec., index error −2′ 0″, and the height of the eye 12 feet; required the longitude.

On May 1 at noon, the chronometer was too fast on Greenwich mean time 1m. 15sec., and its daily rate was 3·2sec. losing (1865).

May 15, Dec.......... = 18° 53′ 41″·3 N., H.D. +35·38.
,, Eq. of time = −3m. 53·49sec., H.D. − ·020
Semi-diameter,...... = 15′ 50″·9.

Ans. Longitude 180° E. or W.

12. March 12, 1874, at 2h. 57m. P.M. (mean time nearly) in latitude 37° 30′ N., and longitude 23° 50′ W. (by account), when a chronometer showed 4h. 43m. 30sec., the observed altitude of the sun's lower limb was 33° 55′ 50″, the index correction was −2′ 5″, and the height of the eye above the sea 20 feet; required the longitude.

On March 1 at noon, the chronometer was fast on Greenwich mean time 9m. 22sec., and its daily rate was 3·5sec. gaining (1863).

March 12, Dec.......... = 3° 16′ 5″·3 S., H.D. = −59″
,, Eq. of time = +9m. 54·34sec., H.D. − ·681
Semi-diameter .... = 16′ 7″·3

Ans. Longitude 23° 56′ 45″ W.

13. March 5, 1874, at 3h. 40m. P.M. (mean time nearly), in latitude 30° 20′ S., and longitude by account 15° 29′ E., when a chronometer showed 2h. 59m. 30sec., the observed altitude of the sun's L. L. was 35° 20′ 40″, the index correction was + 3′ 10″, and the height of the eye above the sea 18 feet; required the longitude.

On February 13 at noon, the chronometer was fast on Greenwich mean time 20m. 30sec., and its daily rate was 3·7sec. gaining (1863).

March 5d, Dec.......... = 5° 59′ 59″·3 S., H.D. −57″·93
,, Eq. of time = +11m. 40·85sec., H.D. − ·580
Semi-diameter .... = 16′ 9″·2.

Ans. 15° 23′ 55″ E.

14. August 23, 1874, at 9h. 40m. P.M. (mean time nearly), in latitude 30° 10′ N., and longitude 23° 5′ W. by account, when a chronometer showed 11h. 4m. 20sec., the observed altitude of α Pegasi E. of meridian was 43° 32′ 20″, the index correction was +2′ 2″, and the height of the eye above the sea 20 feet; required the longitude.

On August 1 at noon, the chronometer was slow on Greenwich mean time 5m. 40sec., and its daily rate was 3·7sec. losing (1863).

Aug. 23, Dec. of Markab = 14° 31′ 52″·1 N.
,, R.A. ,, = 22h. 58m. 31·66sec.
,, Sidereal time... = 10h. 6m. 26·24sec.

Ans. Longitude 23° 5′ 10″ W.

15. Prove the formula by which the calculation in the above question is made (1863).

16. May 16, 1874, before noon, in latitude 35° 10′ 40″ N., when a chronometer showed 4h. 10m. 21sec., the observed altitude of the sun's lower limb was 38° 59′ 50″, index error −2′ 25″, height of the eye 17 feet; required the longitude.

On April 30 the chronometer was slow on Greenwich mean time 1h. 3m. 51sec., and gaining 3sec. per day (1861).

May 16, Dec.......... = 19° 7′ 40″·7 N., H.D. +34″·57
,, Eq. of time = −3m. 52·72sec., H.D. −·044
Semi-diameter ...... = 15′ 50″·7

Ans. Longitude 24° 14′ 15″ W.

17. May 24, 1874, about 1h. 35m. A.M., in latitude 12° 17′ N. and longitude by account 35° 20′ W., when a chronometer showed 4h. 50m. 12sec., the altitude of Arcturus W. of the meridian was 38° 22′ 20″, the index error of the sextant minus 53″, and the height of the eye 18 feet.

On May 8, at Greenwich mean noon, the chronometer was fast on Greenwich mean time 51m. 18sec., and gaining 1·5sec. daily. Find the longitude (1862).

R.A. of Arcturus .... = 14 9 56·74
Dec. ,, ... = 19 50 14·6 N.
Sidereal time, May 23d. .... = 4 3 42·94

Ans. Longitude 35° 14′ 17″ W.

18. What is the use of the chronometer? How is its rate determined? and how is it applied to find differences of longitude?

A chronometer is fast 1h. 12m. 30·5sec. at a place in longitude 45° 18′ 38″ E.; what is its error for a place in longitude 12° 50′ 18″ W. (1864)? *Ans.* 5h. 5m. 6·2sec. fast.

19. On March 7, 1874, the ship's apparent time at a place in longitude 52° 40′ W. was found to be 20h. 25m. 10sec., and the

chronometer showed 11h. 29m. 40sec.; what was the error of the chronometer on mean time at place and on mean time at Greenwich?

On February 20 its error was 1h. 7m.; show what it will be on April 10 (1869).

Greenwich Date.				Longitude in Time.
	h.	m.	sec.	
March 7d.,	20	25	10	52° 40′ W.
	3	30	40	4
G.A.T. ,, 7d.,	23	55	50	6,0)21,0 40
				3h. 30m. 40sec.

Equation of Time.			H.D.	
	m.	sec.		
March 8d. =	10	57·19	$4\frac{1}{16}$	·628
		·04		·042
	+10	57·23	$10\frac{1}{24}$	·001
				·043

		h.	m.	sec.
Greenwich App. Time,	March 7d.,	23	55	50
Equation of Time, ...		0	10	57·23
Greenwich Mean Time,	March 8d.,	0	6	47·23
Chronometer shows, ...	March 7d.,	23	29	40
Chronometer is therefore slow			37	7·23

	h.	m.	sec.	
On February 20d. the error was	1	7	0	slow
On March 7d. ,, is		37	7·23	slow
∴ in 15 days it has gained......		29	52·77	

$$\therefore \text{ in 1 day it has gained } \frac{29\ 52{\cdot}77}{15} = \overset{*\ \text{m.}}{1}\ \overset{\text{sec.}}{59{\cdot}51}$$

∴ Rate 1m. 59·51sec. gaining

* thus 29 52·77
4
6,0)11,9 31·08
1m. 59·51sec.

	m.	sec.	
In 34 days more up to April 10d. it will gain	1	59·51	× 34
		34	
	1h. 7m.	43·34sec.	
On March 7d. it was slow	37	7·25sec.	
Its error on April 10d. is	30	36·09 fast.	

20. May 16, at 6 A.M. nearly, in latitude 42° 37′ N., longitude 115° 30′ W., the observed altitude of the sun's upper limb per artificial horizon was 27° 45′ 20″, the index correction +3′ 12″, the chronometer showed 2h. 10m. 7·5sec.; what is the error of the chronometer (1864)?

May 16, Dec.......... =	19° 7′ 40″.7 N.,	H.D. =	+34″·57
,, Eq. of time =	−3 52″·72,	H.D. =	−·044
Semi-diameter...... =	15 50″·7		

Ans. 27m. 48·5sec. fast.

21. What do you mean by the rate of a chronometer? Explain the ordinary method used by navigators for finding the error and rate of a chronometer.

22. At a place in latitude 53° 20′ S., what is the apparent time when the sun's altitude is 18° 30′, his declination being 10° 10′ S. (1870)? *Ans.* 4h. 50m. 25sec. W., or 19h. 9m. 35sec. E.

23. Show how to find the hour angle or meridian distance of any celestial object from the observed altitude. How is it determined whether it is east or west of the meridian (1863)?

24. Investigate an expression for finding the hour angle of a heavenly body from the observed altitude (1864).

25. Prove the rule for finding the longitude by chronometer (1866).

CHAPTER IX.

AMPLITUDES AND AZIMUTHS.

Definition of Amplitude and Azimuth—Their Use—Proofs—Sin. Amplitude = Sin. Dec. × Sec. Latitude—Rules for Amplitudes—Examples and Examination Questions—Azimuths—Proof for Azimuth—Rules—Examples—Illustrations.

99. **The Amplitude** of a celestial object is its distance from the east point of the heavens when it rises, or from the west when it is setting. It is measured on the horizon.

100. **The Azimuth** of a celestial body is its distance from the south point of the heavens in north latitude, and from the north in south latitude, at any time of the day. It is measured along the horizon from south or north towards east or west. Azimuth may be better defined as the angle at the zenith included between a circle of altitude passing through the body and the meridian of the observer.

Amplitudes and *azimuths* are used in Nautical Astronomy for finding the *variation of the compass.*

101. **The Variation of the Compass** has been so fully entered into in the small treatise on *Navigation* in this series of science works, as to render it unnecessary to say more here. The student is referred to what is there said on the subject.

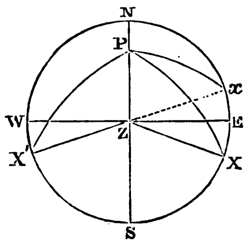
Every position of the sun or celestial object in the heavens corresponds to a certain time, or at any given time the exact bearing of a celestial object can be found. The compass will also show us the bearing of the same object: if the compass bearing and calculated bearing

agree, then the compass has no error; but if they differ, the difference is the error of the compass.

102. Amplitude—*To prove that*

$$\text{Sin. } amplitude = \text{Sin. } declination \times \text{Sec. } lat.$$

When the latitude of a ship and the declination of a heavenly body are known, the amplitude of the body is thus found :—



Let this figure be a projection of the sphere on the plane of the horizon; let P be the position of the pole; Z the position of the zenith; X the position of a sun or star at rising. Then E X or E Z X is the rising amplitude of the object; Z X is its zenith distance = a quadrant; P X is its polar distance, greater than 90° when at X, less than 90° at x.

In the quadrantal triangle P Z X, from Napier's circular parts,

$$\text{Sin. } (90 - \text{P X}) = \text{Cos. } (90 - \text{P Z}).\ \text{Cos. P Z X}.$$
$$\therefore \text{ Cos. P X} = \text{Sin. P Z Cos. } (90 + \text{E X}).$$

But P X is polar dis., and polar dis. = 90 − declination; P Z is the co-latitude; E X is the amplitude, and Cos. (90 + E X) = Sin. E X.

Therefore instead of

$$\text{Cos. P X} = \text{Sin. P Z. Cos. } (90 + \text{E X}).$$

We may write

$$\text{Cos. } (90 - \text{dec.}) = \text{Sin. } co.\text{-}lat. \times \text{Sin. E X}.$$
$$\therefore \text{ Sin. } dec. = \text{Cos. } lat. \times \text{Sin. } amp.$$
$$\therefore \text{ Sin. } amp. = \frac{\text{Sin. } dec.}{\text{Cos. } lat.} = \text{Sin. } dec. \times \text{Sec. lat.}$$

The same thing may as easily be proved from the spherical right-angled triangle P N x, where P N x is the right angle.

$$\text{Sin. } (90 - PZ) = \text{Cos. } PN. \text{ Cos. } Nx$$
$$\therefore \text{ Cos. } PZ = \text{Cos. } PN. \text{ Sin. } Ex$$
$$\text{or Cos. } \textit{polar dis.} = \text{Cos. } \textit{lat.} \times \text{Sin. } \textit{amp.}$$
$$\text{Sin. } \textit{dec.} = \text{Cos. } \textit{lat.} \times \text{Sin. } \textit{amp.}$$
$$\therefore \text{ Sin. amp.} = \text{Sin. dec.} \times \text{Sec. lat.}$$

Rules to find the Amplitude:

(*a*) Find the Greenwich date of rising or setting.

(*b*) Correct the declination.

(*c*) To log. Sin. declination add log. secant latitude, t sum (omitting 10) is log. *Sin.* amplitude, which must marked E. or W. according as the object is rising setting, and N. or S. according to the declination.

(*d*) Observe how far apart these two bearings a asunder, then their distance apart, *i.e.*, the sum if o is north and the other south, their difference if both a north or both south.

(*e*) Mark the amplitude E. if the true is to the right the observed or magnetic, W. if to the left.

(*f*) The deviation is applied thus: if both are E., or bo W., then subtract; if one is east and the other west ad then the variation is E. or W., generally as the greai is *E. or W.*, but the student must examine each ca separately.

1. On January 24d., 1874, in latitude 38° 42′ 5″ N., longitu 9° 8′ W., the sun rose by compass E. 21° S. at 7h. 6m. A. mean time; required the variation of the compass.

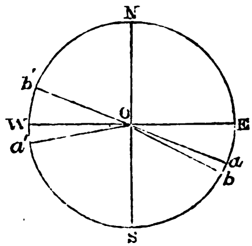
Greenwich date.	h.	m.	sec.		Longitude in time.
January 23d.	19	6	0		9° 8′ W.
Long.		36	32	W.	4
23d.	19	42	32		6,0)3,6 32
					36m. 32sec.

Declination.		
January 24d. = 19° 10′ 29″·8 S.	= 36″·32	6,0)28
+2 35·8	4·29	6,0)17·
19° 13′ 5·6	32688	4·
	7264	
	14528	
	6,0)15,5·8128	
	2 35·8	

Sin. *amplitude* = Sin. *declination* × Sec. *latitude*.

Dec. 19° 13′ 6″	.... Sin.	9·517418
Lat. 38 42 30	.... Sec.	10·107716
True amp. E. 24° 57′ S.	Sin.	9·625134
Obs. amp. E. 21° 0′ S.		
Variation 3° 57′ E.		

The 24° 57′ is marked S. because the declination is S.; and E. because it is a rising amplitude. They are subtracted to find the angle a O b.



The 3° 57′ is marked E. because by examining the figure, if O a be the observed amplitude, and O b the true, we see that the true Ob is to the right of O a when we stand in centre of the compass.

2. On 16th August, 1874, at 7h. 5m. mean time at place, the sun set W. 16° S. in latitude 46° 40′ N., longitude 53° 3′ W., what is the variation of the compass?

Greenwich date.

	h. m. sec.
Aug. 16d.	7 5 0
Long.	3 32 12
16d.	10 37 12

Longitude in time.

53° 3′ W.
4
6,0)21,2 12
3 32 12

Declination.

Aug. 16d. = 13° 43′ 54″·5 N.
−8 24
13 35 30·5 N.

H. D.

−47″·47
10·62
9494
28482
4747
6,0)50,4·1314
8′ 24″

6,0)12
6,0)37·2
10·62

Sin. *amplitude* = Sin. *dec.* × Sec. *lat.*

Dec.	13° 35′ 30″	Sin.	9·371069
Lat.	46 40 0	Sec.	10·163523
True amp. W.	20° 1′ N.	Sin	9·534592
Obs. amp. W.	16 0 S.		
Variation	36 1 E.		

The 20° 1′ is marked N. because the declination is N., and W. because it is a setting amplitude; the two are added together to find the variation, for it is evident from figure that the sum is the difference of the bearings or the angle a' O b'.

The variation 36° 1′ is marked E. because the *true* O b' is to the right of the observed O a' to a person standing in the centre of the compass.

3. On May, 1d. 1874, 6h. 50m. A.M. mean time, in latitude 48° 17′ N., longitude 30° W., the amplitude of Spica when rising is observed to be E. 41° 10″ S., required the variation.

Dec. of Spica 10° 30′ 22″·8 S.

Sin. *amplitude* = Sin. *dec.* × Sec. *lat.*

Declination	10°	30′	22″·8		Sin.	9·260894
Latitude	48	17	0	N.	Sec.	10·176886
True amp. E.	15°	54′		S.	Sin.	9·437780
Obs. amp. E.	41	10		S.		
Variation	25	16		W.		

Precisely as we have done in the last figure to show the position of the true and observed bearings of the sun in each of the two examples, so the student must in every case with his amplitude. *He must draw the figure, and then there can be no mistake.* But instead of marking them b and b' or a and a', it is more advisable to mark them *true* and *obs.* Then at a glance, supposing the position of the spectator to be in the centre of the compass, it is seen whether the true is to the right or left of the observed.

All the following examples are taken from examination papers :—

4. Define amplitude and azimuth. For what practical purpose are the amplitude and azimuth observed at sea?

5. January 20, 1874, in latitude 47° 8′ S., longitude 110° E., about 5h, 40m. mean time, the sun rose by compass E. 10° 20′ S., deviation 4° 50′ E.; required the variation of the compass (1870).

January 19d. Dec. = 20° 18′ 36″·4 S., H.D. − 31″·74

Ans. Var. and dev. 20° 29′ E.
Variation 15° 39′ E.

6. August 31, 1874, at 6h. 26m. A.M., in latitude 35° 60′ S., longitude 80° 10′ E., the sun rose by compass due east (ship's head E., deviation 8° 50′ E.); required the variation (1871).

Greenwich date.		Longitude in time.
	h. m. sec.	
Aug. 30d.	18 26 0	80° 10′ E.
	5 20 40	4
30d.	13 5 20	6,0)32,0 40
	10 54 40	5 20 40

Declination.

	H.D.		
Aug. 31d. = 8° 37′ 34″·1 N.	30	½	54·″13 –
+9 50·6			10
8 47 24·7			541·3
	20	⅓	27·06
	4	⅕	18·04
	40	⅙	3·60
			·60
		6,0)	59,0·6
			9′50″·6

Sin. amp. = Sin. dec. × Sec. lat.

	Sin. 8° 47′ 25″*d.*	=log.	9·184174
	Sec. 35 50 0 *l.*	=log.	10·091127
Sin. true amp.	E. 10 52 N.	=log.	9·275301
Obs. amp.	E. 0 0 N.		
Var. and dev.	10 52 W.		
Deviation	8 50 E.		
Variation	19 42 W.		

7. State and prove the rule for finding the variation of the compass by observing the sun's azimuth (1867).

8. May 19, 1874, at 4h. 50m. A.M., in latitude 19° 1′ S., longitude 37° 18′ E., the sun rose by compass E. 21° 18′ N., the ship's head N.E. (deviation 10° E.); required the variation.

May 19d., Declination 19° 47′ 41″·4 N., H.D. = +32′ 10″

Ans. Var. and dev. 0° 24′ E.
Variation 9° 36′ W.

9. Prove the rule employed in the solution of the last question (1866).

10. May 15, 1874, at 4h. 20m. A.M., in latitude 23° 5′ S., longitude 20° 13′ W., the sun rose by compass E. 25° 23′ N., the

ship's head being E. (deviation 8° E.); required the variation (1867).

May 15d. Dec. = 18° 53′ 41″·3 H.D. + 35″·38

Ans. Var. and dev. 4° 50′ E.
Variation 3° 10′ W.

11. State and prove the rule for finding the variation by an observed amplitude of the sun (1868).

12. September 6, 1874, at 5h. 29m. P.M., in longitude 115° 10′ E., lat. 58° 10′ S., the sun set by compass due west (ship's head S.S.W.), deviation 5° W.); required the variation.

September 6d. Dec. = 6° 25′ 16″·2 N. H.D. − 56″

Ans. Var. and dev. = 12° 18′ E.
Variation = 17 18 E.

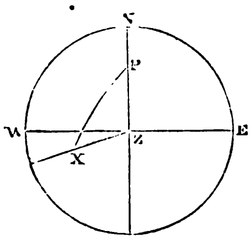
13. Prove the rule applied in the solution of the above question (1865).

14. Define amplitude, and show how to find the variation of the compass by an observed amplitude of the sun. Prove the formula you employ (1869).

103. Azimuths.—Azimuth has already been defined as the angular distance of a heavenly body from the south in north latitude, and from the north in south latitude.

We have *altitude* azimuths and *time* azimuths.

An altitude zimuth is to find the true azimuth when we take the bearing of an object by compass and its altitude at the same time, then from these elements, with the latitude and polar distance, the true bearing is found.



A *time azimuth* is to find the true azimuth by taking the bearing, and then noticing the exact Greenwich time. From this the apparent time or hour angle is found, which, with the polar distance and co-latitude, will give the true bearing.

We have only now to deal with altitude azimuths.

Let this be a projection on the plane of the horizon. Then S Z X is the azimuth if X be the position of the object, P and Z the pole and zenith.

By spherical trigonometry

$$\text{Cos. P Z X} = \frac{\text{Cos. P X} - \text{Cos. P Z Cos. Z X}}{\text{Sin. P Z. Sin. Z X}}$$

but Cos. P Z X = − Cos. S Z X or − Cos. Azim.

$$\therefore\ -\text{Cos. Azim.} = \frac{\text{Cos. } p. - \text{Cos. } co\text{-}lat. \times \text{Cos. zen. dis.}}{\text{Sin. co-lat.} \times \text{Sin. zen. dis.}}$$

$$-\text{Cos. Azim.} = \frac{\text{Cos. } p. - \text{Sin. lat.} \times \text{Sin. alt.}}{\text{Cos. lat.} \times \text{Cos. alt.}}$$

add 1 to each side.

$$\therefore\ 1 - \text{Cos. Azim.} = \frac{\text{Cos. } l \text{ Cos. } a - \text{Sin. } l. \text{ Sin. } a + \text{Cos. } p.}{\text{Cos. } l. \text{ Cos. } a.}$$

$$\therefore\ 2 \text{ Sin.}^2 \frac{\text{Azim.}}{2} = \frac{\text{Cos. } (a+l) + \text{Cos. } p.}{\text{Cos. } l. \text{ Cos. } a.}$$

$$= \frac{2 \text{ Cos. } \frac{a+l+p}{2} \cdot \text{Cos. } \frac{a+l-p}{2}}{\text{Cos. } l \cdot \text{Cos. } a}$$

$$\therefore \text{Sin. Azim.} = \left\{ \frac{\text{Cos. } \frac{a+l+p}{2} \cdot \text{Cos. } \frac{a+l-p}{2}}{\text{Cos. } l. \ \text{Cos. } a} \right\}^{\frac{1}{2}}$$

$$\text{or log. Sin. Azim.} = \tfrac{1}{2} \left\{ \text{log. Cos.} \left(\frac{a+l+p}{2}\right) + \text{log. Cos. } \frac{a+l-p}{2} \right.$$
$$\left. - \text{log. Cos. } l - \text{log. Cos } a \right\}$$
$$= \tfrac{1}{2} \left\{ \text{log. Cos. } \frac{a+l+p}{2} + \text{log. Cos.} \frac{a+l-p}{2} \right.$$
$$\left. + \text{log. Sec. } l + \text{log. Sec. } a \right\}$$

104. Amplitude.—Another proof.

Taking the equation, we started with

$$\text{Cos. P Z X} = \frac{\text{Cos. P X} - \text{Cos. P Z. Cos. Z X}}{\text{Sin. P Z. Sin. Z X}}$$

P Z X being the azimuth, then 90 − P Z X is the amplitude from E. or W., the other quantities will remain as before, but in an amplitude the sun, etc., is on the horizon, and, therefore, the altitude is O $\therefore$ Cos. $a = 1$ and Sin. $a = \text{O}$, so the equation

$$\text{Cos. Azim.} = \frac{\text{Cos. } p - \text{Sin. } l. \text{ Sin. } a}{\text{Cos. } l. \ \text{Cos. } a} \quad \text{becomes}$$

$$\text{Cos. } (90 - \text{amp.}) = \frac{\text{Cos. } p}{\text{Cos. } l} = \frac{\text{Sin. dec.}}{\text{Cos. lat.}}$$

$$\therefore \text{ Sin. } amp. = \text{Sin. } dec. - \text{Sec. lat.}$$

To resume the Azimuth.

Rules for finding the Azimuth of a celestial body.

(1) Find the Greenwich date.

(2) Correct the declination, and find the polar distance.

(3) Correct the altitude.

(4) Put down in order polar distance, latitude, and altitude; take their sum and half sum; take the difference between the polar distance and half sum; next add together log. Sec. *lat.*, log. Sec. *alt.*, log. Cos. *half sum*, log. Cos. *diff.* of p and half sum; divide the sum of these logs. by *two*, and take out the sine of the result. Twice this is the true azimuth.

(5) Having found the true azimuth, if in north latitude, call it S.; if in south latitude, call it N.; if in the morning, call it E.; if in the afternoon, W.

(6) Find the variation of the compass as in the case of an amplitude.

1. On June 12d., at 3h. 10m. P.M. mean time at ship, in latitude 34° 10′ S., longitude 18° 20′ E., the sun's magnetic amplitude was N. 49° W., the altitude of the sun's lower limb was 16° 49′ 37″, index error − 1′ 17″, height of eye 16 feet; required the azimuth and the error of the compass.

Greenwich Date.

	h.	m.	sec.
June 12d.	3	10	0
	1	13	20
12d.	1	56	40

Longitude in Time.

18° 20′ E.
4
6,0)7,3 20
1h. 13m. 20sec.

Declination.

	23°	10′	14″·8 N.
			+18·2
Dec.	23	10	33
	90		
Polar dis.	113	10	33

H.D.

+9·41
1·944
3764
3764
8469
941
18·29304

6,0)40
6,0)5,6·66
·944

Obs. alt. =	16°	49′	37″
Error		− 1	17
	16	48	20
Dip.		− 3	54
	16	44	26
Semi		15	46·9
	17	0	13
Ref.		− 3	1
	16	57	12
Para.			+ 8
True alt.	16	57	20

$$\text{Sin. azimuth} = \left\{ \frac{\text{Cos. } \frac{a+l+p}{2} \text{ .Cos. } \frac{a+l-p}{2}}{\text{Cos. } l. \quad \text{Cos. } a.} \right\}^{\frac{1}{2}}$$

To calculate true azimuth and compass error.

p =	113°	10′	33″		
l =	34	10	0	Sec.	10·082281
a =	16	57	20	Sec.	10·019300
2)	164	17	53		
	82	8	56	Cos.	9·135451
	113	10	32		
	31	1	36	Cos.	9·932945
					2)19·169977
Half azimuth..........	22°	37′	4″	Sin.	9·584988
			2		
True azimuth.......N.	45	14	8 W.		
Obs. azimuth.......N.	49	0	0 W.		
Variation of compass	3	45	52 E.		

2. On November 10 day, at 10h. 15m. 20sec. A.M. mean time at Port Louis in the Mauritius, latitude 20° 10′ N., longitude 57° 31′ 45″ E., the sun's magnetic azimuth was S. 41° 38′ 30″ E., the altitude of the sun's lower limb was 46° 35′ 30″, index correction +2′ 17″, height of eye 19 feet; required the azimuth and error of the compass.

Greenwich date.

	h. m. sec.
Nov. 9d.............	22 15 20
Longitude...........	3 50 7 E.
G. M. T. Nov. 9d.	18 25 13

Longitude in time.

57° 31′ 45″
4
6,0)23,0 7 0
3h. 50m. 7sec.

	Declination.	H. D.
Nov. 10d. =	17° 11′ 43″·2 S.	+42″·14
	3 55 1	5·58
Cor. dec.	17 7 48	33712
	90	21070
p d...........	107 7 48	21070
		6,0)23,5·1412
		3′ 55·″1

To correct the altitude.

Obs. alt.	46° 35′ 30″
Index	+2 17
	46 37 47
Dip	−4 15
	46 33 32
Semi-diameter	+16 11·8
	46 49 44
Ref.	−54
	46 48 50
Para.	+6
True alt.	46 48 56

To calculate the azimuth and compass error

	p...107° 7′ 48″		
	l... 20 10 0	Sec.	10·027476
	a... 46 48 56	Sec.	10·164722
	2)174 6 44		
	87 3 22	Cos.	8·710611
	107 7 48		
	20 4 26	Cos.	9·972782
			2)18·875591
Half azimuth....	15° 54′ 15″	Sin.	9·437795
	2		
True azimuth	S. 31 48 30 E.		
Obs. azimuth	S. 41 38 30 E.		
Variation.........	9 50 0 E.		

Suppose the deviation has been, for the position of the ship's head, 4° 12′ W.,

The variation is found thus :

Deviation and variation together	=	9° 50′ E.
Deviation alone......................	=	4° 12′ W.
∴ Variation alone......................	=	14° 2′ E.

3. On February 9d., 1874, at 9h. 10m. 20sec. mean time at Plymouth, latitude 50° 22′ 25″ N., longitude 4° 7′ 15″ W., the magnetic azimuth of ι Hydræ was S. 23° 11′ W., its altitude was 38° 55′ 20″, index error −4′ 17″, height of eye 54 feet; required the variation of the compass.

Dec. of ι Hydræ, 6° 52′ 47″ N; Polar distance, 83° 7′ 13″.

To correct the altitude.

Obs. alt.	38°	55′	20″
Error....		−4	17
	38	51	3
Dip......		−7	10
	38	43	53
Ref.		1	12
	38	42	41

To calculate the azimuth and find compass variation.

p...	83°	7′	13″		
l ...	50	22	25	Sec.	10·195329
a...	38	42	41	Sec.	10·107735
2)	172	12	19		
	86	6	9	Cos.	8·832330
	83	7	13		
	2	58	56	Cos.	9·999412
					2)19·134806
	21°	40′	25″	Sin.	9·567403
			2		
True azimuth S.	43	20	50 W.		
Obs. azimuth S.	23	11	0 W.		
Variation.........	20	9	50 E.		

Now suppose the deviation has been, for the position of the ship's head, 10° 8′ W.,

We find the variation thus:

Deviation and variation together = 20° 10′ E.
Deviation alone...................... = 10° 8′ W.
∴ Variation alone...................... = 30° 18′ E.

EXERCISES CHIEFLY FROM EXAMINATION PAPERS.

(See remarks preceding the exercises in longitude.)

4. Prove the rule for finding the true azimuth of the sun,

having given the time, the altitude, and declination of the sun, and the latitude.

How can this be employed to obtain the variation of the compass (1871)?

5. On May 14, 1874, in latitude 46° 30′ N., longitude 67° 30′ E., the variation of the compass is 16° 20′ W.; wishing to find the deviation when the ship's head is N.E. about 8h. 30m. A.M. mean time, I observe the sun bears by compass N. 84° 10′ E., when the observed altitude of his L. L. is 14° 10′ 36″, index cor. – 4′ 10″, height of eye 20 feet (1870).

Greenwich date.			Longitude in time.
	h.	m.	
May 13d.	20	30	67° 30′ E.
	4	30 E.	4
13d.	16	0	6,0)27,0 0
			4h. 30m.

Declination.	H. D.		
14d. = 18° 39′ 22″·7 N.	+36·17	*a*	14° 10′ 36″
4 49·4	8	Error......	–4 10
18 34 33·3	6,0)28,9·36		14 6 26
90	4′ 49″·36	Dip	4 28
p = 71 25 27			14 1 58
		Semi.	15 51
			14 17 49
		R. and P.	3 33
			14 14 16

p = 71° 25′ 27″		
l = 46 30 0	Sec.	10·162188
a = 14 14 16	Sec.	10·013549
2)132 9 43		
66 4 51	Cos.	9·607936
71 25 27		
5 20 36	Cos.	9·998109
		2)19·781782
Half azimuth 51° 3′	Sin...	9·890891
2		

S. 102 6 E. = N. 77° 54′ E.
N. 84 10 E. = N. 84 10 E.
Variation and deviation...... 6 16 W.
Variation........................ 16 20 W.
∴ Deviation........................ 10 4 E.

6. January 26, 1874, at 7h. 30m. A.M., in latitude 45° S., longitude 150° W., the sun bore by compass due east (ship's head

E. by N., deviation 9° 55′ E.), when the observed altitude of sun's lower limb was 17° 2′ 10″, the index correction − 1′ 35″, and the height of the eye 12 feet; required the variation (1871).

January 26, Declination...... = 18° 40′ 45″·2 S. H. D. = −38″·04
Semi-diameter = 0° 16′ 16″·6

Ans. Variation and deviation 9° 36′ E.
Variation.................. 19′ W.

7. October 15, 1874, at 3h. 46m. P.M., in latitude 50° 12′ N., longitude 160° 50′ E., the sun bore by compass W. 10° 10′ S. (ship's head E. by N., deviation 11° 30′ E.), when the observed altitude of the sun's L. L. was 11° 29′ 30″, the index cor. + 3′ 55″, and the height of the eye above the sea 14 feet; required the variation (1868).

October 15d., Declination...... = 8° 33′ 53″·5 S. H. D. + 55″·59
Semi-diameter = 16′ 5″·3

Ans. Variation and deviation 18° 44′ W.
Variation.................. 30° 14′ W.

State and prove the rule for finding the variation of the compass by observing the sun's azimuth (1867).

8. February 20, 1874, at 9h. 30m. A.M. mean time, nearly in latitude 43° 20′ N., and longitude 17° 30′ W., the observed altitude of sun's L. L. was 23° 10′, index error − 3′ 20″, height of the eye 24 feet, the sun bore by compass S. 67° 50′ E., and the deviation was 5° 10′ W.; required the variation of the compass (1869).

February 20d., Declination..... = 10° 52′ 3″·9 S. H. D. − 53″·94
Semi-diameter = 0° 16′ 12″·2

Ans. Variation and deviation 21° 20′ E.
Variation .... 26° 30′ E.

CHAPTER X.

THE MOON.

Distance of Moon—Semi-diameter of Moon—Augmentation of Moon's Semi-diameter—Proof—Moon's Declination—Rules to Correct Moon's Declination—To Correct Semi-diameter and Horizontal Parallax—Rules—Right Ascension of Moon, and Rules to Correct it—Moon's Meridian Passage and Retardation—Latitude by Meridian Altitude of the Moon—Examples—Rules—Examination Questions.

THE distance of the moon from the earth is 238,828 miles, or 30·1367 times the earth's diameter. She appears much larger than other celestial objects, because she is, comparatively speaking, so very near us. Her diameter is 2165 miles, while she is about $\frac{1}{50}$ of the size of the earth. Her parallax varies from 29′ 32″ to 33′ 31″. She moves round the earth at a velocity a little greater than 38 miles per minute, taking 27·3216 mean solar days to complete one revolution; as she revolves on her axis once in a complete revolution, her length of day is the same, 27·3216 of our mean solar days. Her orbit inclines to the ecliptic at an angle of 5° 9′.

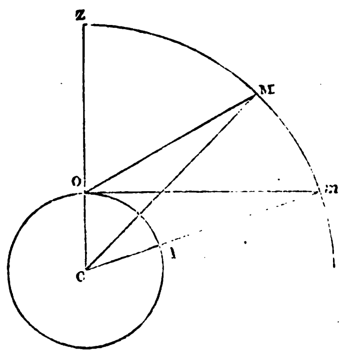
In appearance the moon varies in size, because at one period she is nearer the earth than at another; during a total eclipse she just covers the sun; during an annular eclipse she does not quite cover the sun, but leaves a bright ring all round; this simple illustration shows that in appearance she varies in size.

The exact time elapsing between two consecutive new or full moons is 29d. 12h. 44m. 2·87sec. = 29·5306 days; this is because the earth has moved on in its orbit, so to

reach her proper place for completing a revolution, she has to move on. Were the earth still, the period would be 27d. 7h. 43m. 11·5sec. = 27·3216 days; this is its sidereal period of revolution. A *lunation* or *synodical* period is 29·5306 days.

105. Semi-diameter of Moon.—The semi-diameter of the moon is half the angle subtended by the apparent diameter at the eye of the observer.

The true altitude of the sun, moon, or planets, is the angular perpendicular distance, if we may use such an expression, of their centres from the horizon. Since it is impossible to judge with any degree of accuracy when the object is exactly bisected by the horizon, it is usual with observers "to bring the object down" until its upper or lower limb sweeps the horizon, and to such observations the semi-diameter, as given in the *Nautical Almanac* for each day, has to be applied. It is subtracted if the limb observed is the upper, and added if it be the lower. Upper limb observations are sometimes marked thus $\overline{O}$, lower limb thus $\underline{O}$. When the moon is in the horizon at *m*, its distance from an observer on the surface of the earth at O, is greater than when it is at M. The distance of the moon is not reckoned from the earth's surface, but from its centre. From a spectator at A the moon would be distant 59 times the radius of the earth, while from one at C it would be distant 60 times its radius. Two observers, one at O, the other at A, would see the moon, *m*, differently; to A it would be in his zenith, to O it would be on the horizon; but the spectator at A would



see it 59 times the radius away, while the one at O would see it farther away than this; hence, as the moon gets higher, its diameter will appear increased or augmented. This is what is known as the **Augmentation** of the moon's semi-diameter.

We have shown above that the angular measurement of the semi-diameter of the moon depends upon the position of the spectator, or upon the length of the lines M C and M O.

Let s be the semi-diameter of the moon when at m;
Let $(s+a)$ „ „ „ „ M,
where a is the augmentation due to decrease of distance.

$$\therefore \frac{\text{MC}}{\text{MO}} = \frac{s+a}{s}$$

If MZ or ZOM is the apparent zenith distance z, then C M O is the parallax; call angle C M O...p.

$$\therefore \text{Since MOZ} = \text{MCO} + \text{CMO}$$
$$\therefore \text{MCO} = \text{MOZ} - \text{CMO} = z - p.$$

Now above

$$\frac{s+a}{s} = \frac{\text{MC}}{\text{MO}} = \frac{\text{Sin. MOC}}{\text{Sin. OCM}} = \frac{\text{Sin. MOZ}}{\text{Sin. OCM}} = \frac{\text{Sin. } z}{\text{Sin. } z-p}$$

$$\therefore \frac{s+a}{s} = \frac{\text{Sin. } z}{\text{Sin. } z-p}$$

$$\text{Or, } \frac{s+a}{s} - 1 = \frac{\text{Sin. } z}{\text{Sin. } z-p} - 1$$

$$\therefore \frac{a}{s} = \frac{\text{Sin. } z - \text{Sin. } (z-p)}{\text{Sin. } (z-p)}$$

$$\because \text{Sin. A} - \text{Sin. B} = 2 \text{ Cos. } \frac{\text{A}+\text{B}}{2} \cdot \text{Sin. } \frac{\text{A}-\text{B}}{2}$$

$$\therefore \frac{a}{s} = \frac{2 \text{ Cos. } \frac{2z-p}{2} \cdot \text{Sin. } \frac{z-z+p}{2}}{\text{Sin. } (z-p)}$$

$$\therefore \frac{a}{s} = \frac{2 \text{ Cos. } \left(z - \frac{p}{2}\right) \cdot \text{Sin. } \frac{p}{2}}{\text{Sin. } (z-p)}$$

$$\therefore a \text{ or augmentation} = \frac{2s \text{ Cos. } \left(z - \frac{p}{2}\right) \cdot \text{Sin. } \frac{p}{2}}{\text{Sin. } (z-p)}$$

$$= 2s \text{ Cos. } \left(z - \frac{p}{2}\right) \text{ Sin. } \frac{p}{2} \text{ Cosec. } (z-p).$$

The formula already proved

Para. in alt. = **hoz. parallax × Cos. app. alt.**,

gives the value of p, or parallax in altitude, to be used in the above equation.

To save trouble in correcting an altitude of the moon, a table is constructed from which the student at once takes the augmentation of the moon's semi-diameter; then, when the semi-diameter is corrected, it can be applied to the *observed altitude* to find the *true*.

106. The Moon's Declination.—The moon does not pass over the same path in the sky in each successive year; it has to complete a cycle of 18·6 years before it moves in precisely the same path again. We said that the orbit of the moon inclines to the ecliptic at an angle of 5° 9′, the exact inclination is 5° 8′ 48″. The moon's declination is given on pages V. to XII. of each month for every hour of the day, with the difference for every ten minutes. The declination of the moon is given as it would appear from the centre of the earth.

We have now to show how the moon's declination is corrected.

1. On Sunday, June 21d. 1874, at eight o'clock in the evening, the moon's declination is 2° 32′ 59″·9 N. decreasing, the difference for ten minutes is 138″·34; what is the declination for June 21d. 8h. 24m. 12sec.?

Declination.	Diff. for 10m.	Time.
June 21d. 8h. = 2° 32′ 59″·9 N.	− 138″·34	6,0)1,2
− 5′ 34″·7	2·42	24·2
Cor. dec. ..... = 2° 27′ 25″·2	27668	
	55336	
	27668	
	6,0)33,4·7828	
	5′ 34″·7	

2. The declination of the moon on the 25th June, 1874, at two o'clock in the morning, is 14° 48′ 30″·1 increasing, the difference for ten minutes is 121″·54; required the declination for June 24d. 14h. 15m. 27sec.

Declination.	Diff. for 10m.	Time.
June 24d. 14h. = 14° 48′ 30″·1 S.	+121·54	6,0)2,7
+3′ 7″·7	1·545	15·45
Cor. dec. .... = 14° 51′ 37″·8 S.	60770	
	48616	
	60770	
	12154	
	6,0)18,7·77930	
	3′ 7″·7	

Rules to correct moon's declination—

Having taken out the quantities from the *Nautical Almanac*, reduce the minutes and seconds to decimals of a minute; then shift the decimal place *one* to the left, and multiply the difference by it. Add the difference if increasing, subtract if decreasing.

(If the time be 40·85m., then multiply by 4·085m.; or if 4·5m., multiply by ·45m., etc.)

3. Find the moon's declination for 1874, July 19d. 23h. 18m. 42sec., when at 23h. it is 4° 49′ 11″·3 S. increasing, difference for 10m. being 136·8. *Ans.* 4° 53′ 27″·1 S.

4. Required the corrected declination of the moon for 1874, July 21d. 20h. 25m. 30sec., when at 20h. it is 14° 35′ 48″·5 S. increasing, and difference for 10m. 121″·48. *Ans.* 14° 40′ 58″·2 S.

5. 1874, July 27d. at noon the moon's declination is 27° 36′ S. decreasing, difference for 10m. 23″·41; required the declination for 29m. 21sec. *Ans.* 27° 34′ 51″·3 S.

6. 1874, September 1d. at 9h. 50m. 20sec., required the declination of the moon, it being 20° 17′ 12″·1 N., increasing at 9 o'clock, difference for 10m. = 120″·88. *Ans.* 20° 27′ 20″·5 N.

7. 1874, September 5d. at 21h. 45m. 9sec., required the declination of the moon, it being 26° 33′ 59″·8 N., decreasing at 21h., difference for 10m. = 53″·66. *Ans.* 26° 29′ 47″·5 N.

8. 1874, September 4d. at 11h. 36m. 20sec., required the declination of the moon, it being 28° 6′ 40″·2 N. at 11 o'clock, and decreasing, difference for 10m. ·72″. *Ans.* 28° 6′ 37″·6 N.

9. 1874, December 16d. at 17h. 45m. 30sec., required the declination of the moon, it being at 17h. = 2′ 14″ S. decreasing, difference for 10m. being 165″·78. *Ans.* 10′ 20″·29 N.

107. Semi-diameter of Moon, to Correct.—We have already explained why this correction is required, because

the apparent size of the moon varies. "The *moon's semi-diameter* is the angle under which her semi-diameter would appear if viewed from the centre of the earth, while

108. "**Horizontal Parallax** is the *greatest* angle under which the earth's equatorial semi-diameter would appear if seen from the centre of the moon. The semi-diameter is requisite to obtain the position of the centre from an observation of the moon's *limb*, as in all cases of altitudes or linear distances, the *horizontal parallax* is required for computing the horizontal parallax of the moon at any given latitude on the earth, *considered as a spheroid;* also for finding the parallax in altitude, right ascension, etc., for the purpose of reducing an observation of the moon made on the surface of the earth to what it would be if made at the centre."—*Nautical Almanac.* The semi-diameter and the horizontal parallax of the moon are given on the III. page of each month in the *Nautical Almanac* for noon and midnight of each day.

10. 1874, September 10d., the semi-diameter of the moon at noon is 14′ 55″·8, at midnight 14′ 53″·2, the altitude above the horizon is 50°; required the semi-diameter for September 10d. 5h. 40m.

Difference and Correction.	
Semi-dia. at noon =	14′ 55″·8
,, midnight =	14′ 53″·2
Diff. for 12h. decr.	2·6
	5⅔
or, as 12 : 5⅔ : : 2·6 : 1·22	13·0
	1·7
	12)14·7
Difference in 5⅔h.	1·22

Semi-diameter Required.	
Semi-dia. at noon =	14′ 55″·8
Correction	−1·2
	14′ 54″·6
Aug. for 50°	11·1
Cor. semi-dia.	15′ 5″·7

11. Sept. 23d., 1874, the semi-diameter of the moon at noon is 16′ 17″·3, at midnight 16′ 23″·9, the altitude above the horizon is 65°; what is the moon's semi-diameter for Sept. 23 day 8h. 15m.?

Difference and Correction.		Semi-diameter Required.	
Semi-dia. at noon =	16′ 17″·3	Semi-dia. at noon =	16′ 17″·3
,, midnight =	16′ 23″·9	Correction	+4″·5
Diff. for 12h. incr.	6·6		16′ 21″·8
	8¼	Aug. for 65°	16
or, as 12 : 8¼ : : 6·6 : 4·5	52·8	Cor. semi-dia.	16′ 37″·8
	1·6		
	12)54·4		
Difference for 8¼h.	4·5		

Rules for correcting the moon's semi-diameter :—

(1) From the *Nautical Almanac* take out the moon's semi-diameter at noon and midnight, or midnight and noon, between which the required one lies.

(2) Take the difference of these two, this gives the change in 12 hours.

(3) Knowing the change for 12 hours, find the change for difference of time required, either by proportion or by multiplying by the given number of hours from noon or midnight, and dividing by 12.

(4) Subtract the correction if decreasing, add it if increasing.

(5) Then from the table find the augmentation. The altitude will be found on the left side, and the semi-diameter above the tables. This correction added to the semi-diameter, will give the correct semi-diameter of moon for given time.

12. Find the semi-diameter of the moon for January 10 day, 1874, at 15h. 10m. 45sec., when the semi-diameter at midnight is 14′ 52″·5, and at noon next day 14′ 55″·5. *Ans.* 14′ 53″·3.

13. Find the semi-diameter of the moon for January 31 day, 19h. 18m., when the semi-diameter at midnight is 15′ 5″·7, and at noon February 1 day 15′ 2″·3. *Ans.* 15′ 3″·63.

14. 1874, April 1 day, the semi-diameter at noon is 14′ 43″·6, at midnight 14′ 43″·9.; correct it so as to obtain it at half-past nine in the evening. *Ans.* 14′ 43″·837.

15. April 1 day at midnight the semi-diameter is as given in the last question, on April 2 day at noon it is 14′ 44″·5; what is it on April 1 day 22h. 45m.? *Ans.* 14′ 44″·437.

109. Right Ascension of the Moon.—Right ascension has already been fully explained; with the declination of

the moon is given in the *Nautical Almanac* its R. A. for every hour of the day, but *no difference* is given; still it has to be corrected for any Greenwich date that may be used.

Rule to correct the R. A. of the moon.—(1) Take out the two R. A.'s, between which the given date lies.

(2) Take their difference.

(3) This being the difference for 60 minutes, by proportion or otherwise find the difference for the minutes and seconds required.

(4) Add this to the first R. A.

16. Find the right ascension of the moon for November 7 day 18h. 16m. 30sec. G. M. T.

Difference and Correction.

	h.	m.	sec.
Nov. 7 day, 18h. =	14	10	5·48
19h.	14	11	56·35
		1	50·87

As 60m. : 16½m. : :	0	1	50·87
			16½
		29	33·92
			55·43
	6,0)	30	29·350
			30′·489

R. A. required.

	h.	m.	sec.
18h.	14	10	5·48
16m. 30sec.			30·489
R. A.	14	10	35·969

17. Find the R. A. of moon, when in longitude 80° W. it is 1854, April 8 day, 13h. 20m. 10sec. mean time at Greenwich.

Difference and Correction.

	h.	m.	sec.
April 8 day, 13h. =	18	36	59·76
14h.	18	39	26·23
20m. = $\frac{1}{3}$		2	26·47
10sec. = $\frac{1}{120}$			48·82
			·40
			49·22

R. A. required.

	h.	m.	sec.
13h.	18	36	59·76
20m. 10sec.			49·22
R. A.	18	37	48·98

18. Find the R. A. of the moon, April 1 day 17h. 18m. 45sec., when at 17h. = 12h. 57m. 36·02sec.
when at 18h. = 12h. 59m. 21·38sec. *Ans.* 12h. 58m. 8·945sec.

19. What is the R. A. of the moon for May 8 day 9h. 10m. 10sec., when at 9h. = 21° 6′ 44″·96?
when at 10h. = 21° 9′ 4″·67? *Ans.* 21° 7′ 8″·6.

20. Find the R. A. of the moon for May 24 day 12h. 18m. 45sec., when at 12h. = 11h. 43m. 52·29sec.
when at 13h. = 11h. 45m. 38·04sec.
Ans. 11h. 44m. 25·335sec.

21. Required the correct R. A. of the moon for June 24 day 19h. 12m. 12sec., at 19h. = 14h. 34m. 19·36sec.
at 20h. = 14h. 36m. 14·86sec.
Ans. 14h. 34m. 42·845sec.

To correct the horizontal parallax of the moon:

Correct the horizontal parallax in precisely the same manner as you would a semi-diameter.

22. December 10 day, 1874, the moon's horizontal parallax at midnight is 55′ 39″·9 at noon, next day 55′ 53″·7; find what it is December 10 day 14h. 18m. 36sec. G. M. T.

Difference and Correction.		Horizontal parallax required.	
H. P. at midnight.........	55′ 39″·9	H. P. at midnight	55′ 39″·9
H. P. at noon...............	55′ 53″·7	Correction	+2″·6
Diff. for 12 hours	13″·8	H. P.	55′ 42″·5
	2·31		
	138		
	414		
	276		
	12)31·878		
Diff. for 2h. 18m. 36sec.	2·656		

There is a reduction required from this for the latitude of the place, to be found in the tables, called the reduction of the moon's horizontal parallax. The horizontal parallax requires reducing, because the earth is a spheroid, and not a perfect sphere; but this is a subject that must be treated more fully in our advanced course.

110. Moon's Meridian Passage and Retardation.—The time of the moon's meridian passage is found in the right hand column of the IV. page of the month in the *Nautical Almanac*. It is the Greenwich mean astronomical time to the nearest tenth of a minute at which the moon's centre is on the upper meridian of Greenwich, and is useful to indicate when the latitude may be obtained from an observed meridian altitude of the moon. Occasionally in the column is found no date; this simply

means that the moon does not pass the upper meridian of Greenwich on that day, which is the case once in every lunation, and arises from the circumstance of the lunar day being longer than the mean solar day, and including it now and then between its limits. The lunar day on January 17d., 1874, is longer by 1h. 1·4m. than the solar. It so happens that the moon passes the meridian January 16d. 23h. 24·4m. mean astronomical time at Greenwich, and does not return to the same meridian until January 18d. 0h. 25·8m. The reason why a lunar day is longer than an astronomical day has been already explained.

111. Retardation.—The moon's meridian passage is the mean time of its passage over the meridian of Greenwich. By applying retardation we correct the error arising from the unequal motion of the moon in right ascension. At any place situated in east longitude, the time of transit must take place earlier in proportion because of the moon's motion towards the east, while it will take place later at any place in west longitude. This change of motion, or retardation, is variable, since the daily motion of the moon varies between 40 and 60 minutes of R.A. By subtracting a meridian passage from the one that immediately precedes it or follows it, we get the daily motion of the moon in R.A. for 24 hours or 360°; knowing the whole motion for 360°, it is very easy to find it for any longitude the observer may be in. To find this retardation when the longitude is west, you take the difference between the meridian passage of the given day and the following day, and add; but if the longitude be east, you take the difference between the given day and the preceding, and subtract, after finding the proportional change for the "longitude in." The following two illustrations will show practically what is meant :—

1. Find the Greenwich date of the moon's meridian passage January, 1870, 21d., in longitude 140° W.

		h.	m.
Meridian passage Jan. 21d. is at		3	13·3
,, ,, ,, 22d. ,,		4	3·2
	Retardation for 360°......		49·9

Retardation for 1°...... $\frac{49\cdot 9}{360}$

Retardation for 140°...... $\frac{49\cdot 9\times 140}{360}=19\cdot 4\text{m.}$

Greenwich date of Meridian Passage.	h. m. sec.	Longitude in time.
January 21d.	3 13 18	140° W.
Retardation	+19 24	4
21d.	3 32 42	6,0)56,0
Longitude W.	9 20 0	9h. 20m.
January 21d.	12 52 42	

2. Find the meridian passage of the moon January, 1874, 12d. in longitude 125° 30′ E.

Looking in the *Nautical Almanac* we see the meridian passage in 19h. 41·7m., which is next day, civil time, and so we see how necessary it is to pay attention to the astronomical date. We have, therefore, to take out the one opposite the 11th day, and as the longitude is E. the 10th day also to find their difference from which to calculate the retardation.

	h.	m.
Meridian passage January 10d. is at	18	14·8
,, ,, ,, 11d. ,,	18	56·6
		41·8

Meridian Passage. Greenwich date.	h. m. sec.	Long. in time.	Retardation.
		125° 30′ E.	41″·8
		4	125½
January 11d.	18 56 36		2090
Retardation	−14 34·3	6,0)50,2 0	5016
	18 42 1·7	8h. 22m.	209
Long. in time	8 22 0 E.		6,0)524,5·9
January 11d.	10 20 1·7		6)87 25·9
			14′ 34″·3

Rules for obtaining the Greenwich date for the meridian passage of the moon:—

(1) Take out the meridian passage from the *Nautical*

Almanac for the given day, but if it is after midnight, 12 o'clock, take the passage for the previous day.

(2) Take out also the meridian passage for the day before if it be *east* longitude, or the succeeding day if it be west longitude; and find the difference between the two passages taken out.

(3) Multiply the difference by the longitude, and divide by 360; this is the retardation to be added if in W. longitude, subtracted if E.

(4) Apply the longitude in time.

3. What time will the moon pass the meridian on June 21d., 1874, at a place in longitude 55° E.?

Meridian passage June 20d. = 5h. 22·8m.
,, ,, ,, 21d. = 6h. 2·8m.

Ans. June 21d. 2h. 16m. 41·4sec.

4. At what time will the moon pass the meridian of the Lizard, latitude 49° 57′ 42″ N., longitude 5° 12′ W., on May 14, 1874?

Meridian passage May 13d. = 22h. 34·8m.
,, ,, ,, 14d. = 23h. 29·6m.

Ans. May 13d. 22h. 56m. 23·2sec.

5. Find the time of the moon's upper transit over the meridian of Sydney, longitude 151° 14′ E., on June 12, 1874.

Meridian passage June 10d. = 21h. 16·9m.
,, ,, ,, 11d. = 22h. 12·2m.

Ans. June 11d. 11h. 44m. 2·2sec.

6. At what time will the moon culminate on the meridian of Valparaiso on May 30d., 1874, in longitude 71° 40′ 30″ W.?

Meridian passage May 30d. = 11h. 40·8m.
,, ,, ,, 31d. = 12h. 33·3m.

Ans. May 30d. 16h. 37m. 57sec.

7. At what time will the meridian passage of the moon take place on September 18d., 1874, at a place in longitude 45° 10′ W.?

Meridian passage September 18d. = 5h. 38·7m.
,, ,, ,, 19d. = 6h. 33·5m.

Ans. September 18d. 8h. 46m. 15sec.

8. Find the time of transit of the moon over the meridian of St Helena, longitude 5° 44′ W. on October 28d., 1874.

Meridian passage October 27d. = 14h. 22·1m.
,, ,, ,, 28d. = 15h. 25·1m.

Ans. October 27d. 14h. 46m. 2sec.

9. On October 23d., 1874, in longitude 45° 30′ E., the observed meridian altitude of the moon's L.L. was 50° 25′ 20″ bearing S.,

index error was − 1′ 25″, and height of eye above the sea 12 feet; find the latitude.

	h. m.
Meridian passage October 22d. =	9 43·2
" " " 23d. =	10 33·9
	50·7

Meridian passage in G. M. T. at observer's position, etc.

	h. m. sec.
October 23d.	10 33 54
	− 6 24·4
	10 27 29·6
Longitude E.	3 2 0
G. D. October 23d.	7 25 29·6

Retardation.

$\dfrac{50{\cdot}7 \times 45\frac{1}{2}}{360}$ = 6m. 24·4sec.

Longitude in time.

45° 30′ E.
4
6,0)18,2 0
3h. 2m.

h. m. sec. hours.
7 25 29·6 = 7·42

To correct semi-diameter.

Semi dia. noon = 16′ 38″·4
" mid. = 16 42·3
3·9
7·42
6678
2226
12)28·938
2·41

To correct horizontal parallax.

H.P. noon 60′ 57″·8
" mid. 61 12·3
14·5
7·42
290
580
1015
12)107·590
8·966

Semi-diameter.

16′ 38″·4
+2·4
16 40·8
Aug. 13·6
16 54·4

Hor. para.

60′ 57″·8
8·96
61 6·76
Red... 5
61 1·7
60
3662

Declination of moon.

At 7h. = 1° 57′ 15″·5 N.
7 36
2 4 51·5

Correction.

5	$\frac{1}{2}$	+ 178·82
		2
		357·64
		89·41
30	$\frac{1}{10}$	8·94
		6,0)45,5·99
		7 36

To correct the altitude and find altitude.

Obs. alt. is.........	50° 25′ 20″	
Index error........	−1 25	
	50 23 55	
Dip..................	−3 22	
	50 20 33	
Ref..................	−47	
	50 19 46	
Semi-dia............	+16 54	H.P. 3662=log. 3·563718
App. alt...........	50 36 40	Cos.=log. 9·802487
Para. in alt........	+38 43	2323=log. 3·366205
True alt............	51 15 23	
	90	
Zen. dis.	38 44 37 N.	
Dec..................	2 4 51·5 N.	
Latitude	40 49 28·5 N.	

We will try and thoroughly explain the manner in which this problem is worked, as there are several little difficulties not yet perhaps sufficiently elucidated, and while doing so will give the

Rules for obtaining latitude by the meridian altitude of the moon.

(1) Meridian passage is found as previously explained, or rather the exact time is found at Greenwich when the moon passes the meridian of the observer. The proper correction for retardation must not be omitted.

(2) Correct the semi-diameter and horizontal parallax, being careful to take out in each case the two from the *Nautical Almanac*, between which the required semi-diameter and horizontal parallax lie. The semi-diameter has to be augmented from the tables according to its altitude and apparent size.

The horizontal parallax has also to be reduced. This reduction depending upon the latitude and parallax, is also taken from the tables.* The latitude wherewith to

* The tables used throughout are Riddle's, but the publishers of this work have in the press a volume of tables for the use of students of Navigation and Nautical Astronomy.

do this is obtained roughly; for instance, we see, after applying the semi-diameter, that the altitude is about $51°$, this subtracted from $90°$ leaves $39°$, then $39° + \text{dec} = 41°$, which is the latitude nearly, with this latitude the reduction of the moon's horizontal parallax is found in the tables.

(3) Having found the horizontal parallax, we obtain the parallax in altitude from the formula

para. in alt. = *hor. para.* × Cos. *app. alt.*

Taking out log. H.P., and adding to it log. Cos. *app. alt.*, gives the parallax in altitude, which is always added to bring the moon to its true geocentric position.

(4) The problem is finished as an ordinary meridian altitude.

10. October 20, 1874, in longitude 68° 40′ W., the observed meridian altitude of the moon's upper limb was 59° 18′ 45″ (zenith south), index error + 2′ 17″, height of eye 10 feet above the sea; in what latitude is the observer?

	h.	m.
Merdian passage October 20d. =	8	1·7
,, ,, ,, 21d. =	8	52·8
Retardation for 24h. or 360° =		51·1

∴ Retardation for 68° 40′ = $\frac{51 \cdot 1 \times 68\frac{2}{3}}{360}$ = 9m. 44·8sec.

Greenwich Mean Time at place.

	h.	m.	sec.
Meridian pass. October 20d.	8	1	42
Retardation		9	44·8
	8	11	26·8
Longitude W.	4	34	40
October 20d.	12	46	6·8

Longitude in Time.

68° 40′ W.
4
6,0)27,4 40
4h. 34m. 40sec.

46m. 6·8sec. = ·77 nearly.

To correct semi-diameter.

Semi-dia. mid. 20d. = 16′ 5″·6
,, noon 21d. = 16′ 13″·3
7·7
·77
539
539
12)5·929
·494

To correct horizontal parallax.

H. P. at mid. 20d. = 58′ 57″·9
,, noon 21d. = 59′ 25″·9
28
·77
196
196
12)21·56
1·8

Semi-diameter.

Mid.	16′	5″·6
		·49
	16	6·09
Aug.		14·6
	16	20·69

Horizontal Parallax.

Mid.	58′ 57″·9
	1·8
	58′ 59″·7
Red.	6″·3
	58′ 53″·4
	60
	3533

Declination.

At 12h.	=16° 26′ 56″·8 S.
	−10′ 40
Cor. dec.	=16° 16′ 16″·8 S.

Correction.

−138″·83
4·61
13883
83298
55532
6,0)64,0·0063
10′ 40″

To correct altitude and find latitude.

Obs. alt. is	59°	18′	45″
Index error		+2	17
	59	21	2
Dip.............		−3	5
	59	17	57
Refraction			−34
	59	17	23
Semi-dia.		−16	21
App. alt.	59	1	2
Para. in alt.		+30	18
True alt.	59	31	20
	90		
Zen. dis.	30	28	40 S.
Dec.	16	16	16·8 S.
Latitude	46	44	56·8 S.

H.P. 3533 log.	=3·548144
Cos. log.	=9·711622
1818 log.	=3·259766

EXERCISES FROM EXAMINATION PAPERS.

1. How are the moon's semi-diameter and horizontal parallax to be found for a given ship date?

June 5d., 1874, at 5h. 30m. P.M., in longitude 35° 15′ W. (1866).

	Semi-dia.	Hor. Para.	
Noon	15′ 51″·5	58′ 6″·1	*Ans.* Semi-dia. 15′ 54
Midnight	15 55·4	58 20·4	Hor. para. 58′ 15″·45.

2. How is the mean time Greenwich date of the moon's meridian passage on a given day in a given longitude found?

May 11d., 1874, civil date, in longitude 54° E. (1867).

Meridian Passage, May 9d. is 19h. 12·9m.
,, ,, 10d. is 20h. 3·2m.

Ans. 10d. 16h. 19m. 39·3sec.

3. How is the moon's declination taken out from the *Nautical Almanac* with a given Greenwich date?

July 4d., 1874, at 3h. 15m. 20sec. P.M. in a place, longitude 36° E.; required the declination of the moon (1867).

Dec. July 4d. 0h. = 10° 25′ 9″·3 S. Diff. for 10m. +153″·87.

4. How is the moon's semi-diameter taken out of the *Nautical Almanac* for a given Greenwich date?

Aug. 18d., 1874, at 5h. 30m. A.M., in a place whose longitude is 45° 30′ E.; required the moon's semi-diameter (1868).

Semi-dia., Midnight, Aug. 17d. = 14′ 47″·1
,, Noon, Aug. 18d. = 14′ 48″

Ans. 14′ 47″·28.

5. What is a parallax? How does it affect the apparent position of a heavenly body? What is meant by the augmentation of the moon's semi-diameter (1868)?

6. August 2d., 1874, in longitude 24° 35′ 20″ E., the observed meridian altitude of moon's L. L. was 67° 18′ 30″, zenith S. of moon, index correction +4′ 35″, height of the eye 24 feet; required the latitude (1868).

Mer. Pass. July 31d. = 14h. 41·2m.
,, Aug. 1d. = 15h. 30·3m.
Dec., Aug. 1d. 13h. = 1° 54′ 27″·4 S. Diff. for 10m. = - 168″·55

	Semi-dia.	Hor. Para.
Midnight, Aug. 1d.	16′ 18″·2	59′ 44″
Noon, ,, 2d.	16′ 17″·1	59′ 42″·2

Ans. Lat. 23° 43′ 36″.3 S.

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